MULTIPLICATION AND CHARGE AVALANCHE

Multiplication

- Ionisation gave us just a few electrons+ions per mm of gas. We have transported them to the read-out, hopefully not losing too many.
- But ... if we collect them directly on a read-out electrode, the current will be tiny.
- We need to multiply them.
- Requires fields where the electron energy occasionally is sufficient to ionise.
- 1901: Gas multiplication by John Townsend

▶ John Townsend:

Let a force X be applied to N_0 negative ions in a gas at pressure p and temperature t. Let N be the total number of negative ions after the N_0 ions have travelled a distance x. The new negative ions travel with the same velocity as the original N_0 ions, so that all the negative ions will be found together during the motion. The number of negative ions produced by N ions travelling through a distance dx will be $\alpha N dx$; where α is a constant depending on X, p, and t.
Then

$$dN = \alpha N dx.$$

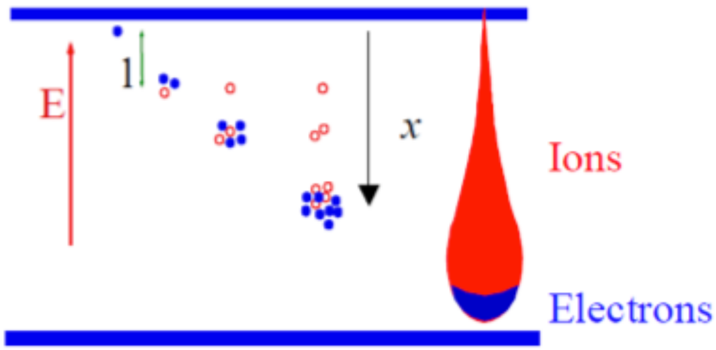
Hence

$$N = N_0 \epsilon^{\alpha x}$$

Sir John Sealy Edward Townsend (1868-1957)

[J.S. Townsend, "*The conductivity produced in gases by the motion of negatively charged ions*", Phil. Mag. **6-1** (1901) 198-227. If access to the Philosophical Magazine is restricted, then consult a German-language abstract at http://jfm.sub.uni-goettingen.de/.]

Avalanche Multiplication



α(E) is determined by the excitation and ionization cross section of the electrons in the gas. It depends also on various and complex energy transfer mechanisms between gas molecules. It has to be measured for every gas mixtures

The electrons produced by primary ionization gain enough energy to create other (secondary) e-ion pairs in secondary ionization events

(Secondary) Electron Energy minimum ~ 20-30eV

- **α : probability for a primary electron to create a secondary electron per unit lenght (Townsend coefficient)**
- **$dn = n_0*\alpha*dx \rightarrow n=n_0*\exp(\alpha x)$**
- **$M=n/n0 = \exp(\alpha x)$ Gain**
  - n0= number of primary electrons
  - Max gain in standard conditions = 10^8 ($\alpha x=20$)
  - After this value, breakdown occurs and a continous discharge affects the detector (Raether limit)
  - The gain per unit lenght depends just on the Townsend coefficient that is determined by the electric field (thus by the applied voltage, once the gas has been fixed)
  - Various theoretical model has been developed to define the dependence of α from the electric field → not triv

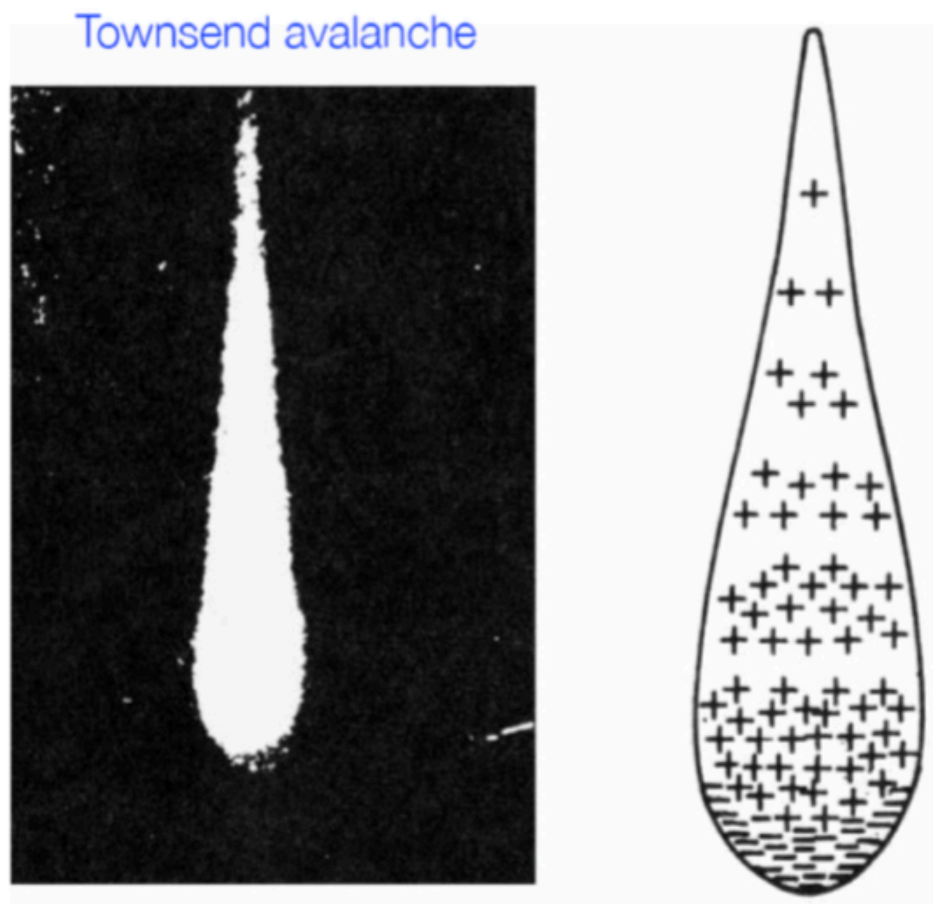
$$M = \exp[\int_{x_1}^{x_2} \alpha(x)dx]$$

Avalanche Multiplication

- The high mobility of electrons results in liquid-drop-like avalanche, with electrons at the head
  - Due to the difference in the mobility of electrons/ions
- Mean free path (to secondary ionization): λ_{ion}
- Drop-like shape of an avalanche
- Probability of an ionization per unit path length:

$\alpha = 1/\lambda_{ion}$ First Townsend coefficient

Townsend coefficient =: probability per unit length that an electron creates an additional electron.



Drop-like shape of an avalanche
Left: cloud chamber picture
Right: hematic view

First Townsend coefficient – Amplification Factor

- Primary electrons in high electric fields reach enough energy to produce ionisation → secondary electrons, they may produce further electrons → avalanche formation
- **The first Townsend coefficient α** is the **number of electron ion pairs** produced by a primary electron per path length:

$\lambda_{ion} = 1/N*\sigma_i$
$\alpha = 1/\lambda_{ion}$

$$\alpha = \sigma_i \cdot \frac{N_A}{V_{mol}}$$

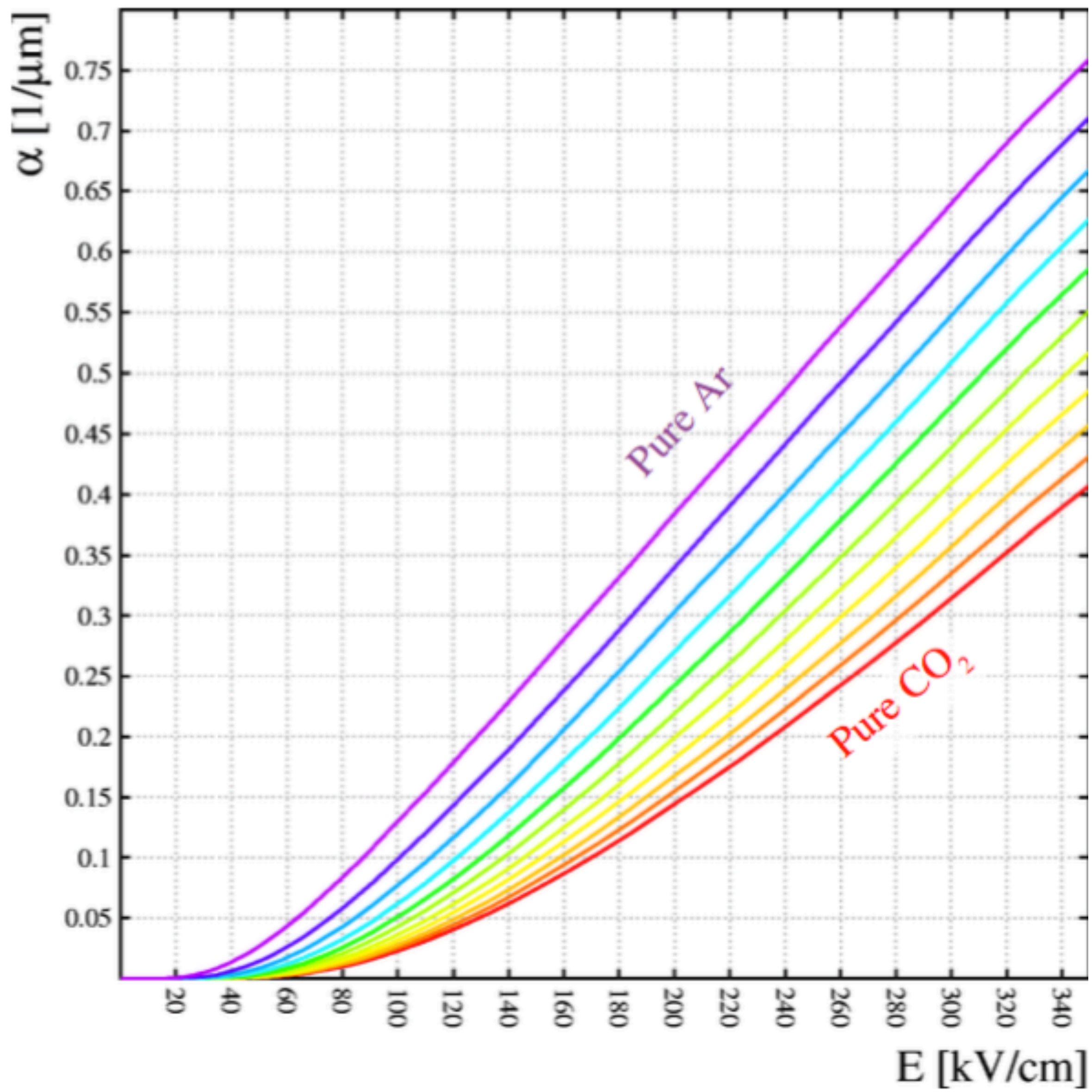
σ_i … Ionization cross section
N_A … Avogadro's number
V_{mol} … Molar volume (ideal gas: 22.4 l/Mol)

- σ_i depends on the electron energy, the electron energy is given by the acceleration in the electric field depending on the position within the detector → **α = α(*x*) depends on position in the detector**
- **α depends on the gas density!**
- Number of produced electrons and amplification factor *A*

$$N(x) = N_0 \cdot e^{\int \alpha(x)\,dx} = N_0 \cdot A$$

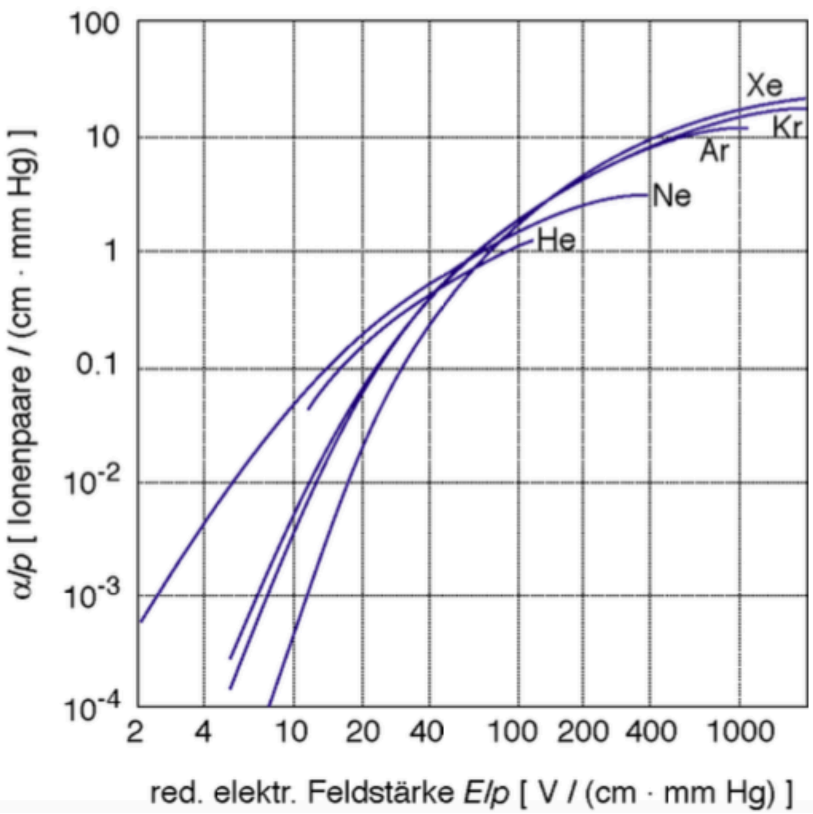
First Townsend α in Ar-CO2

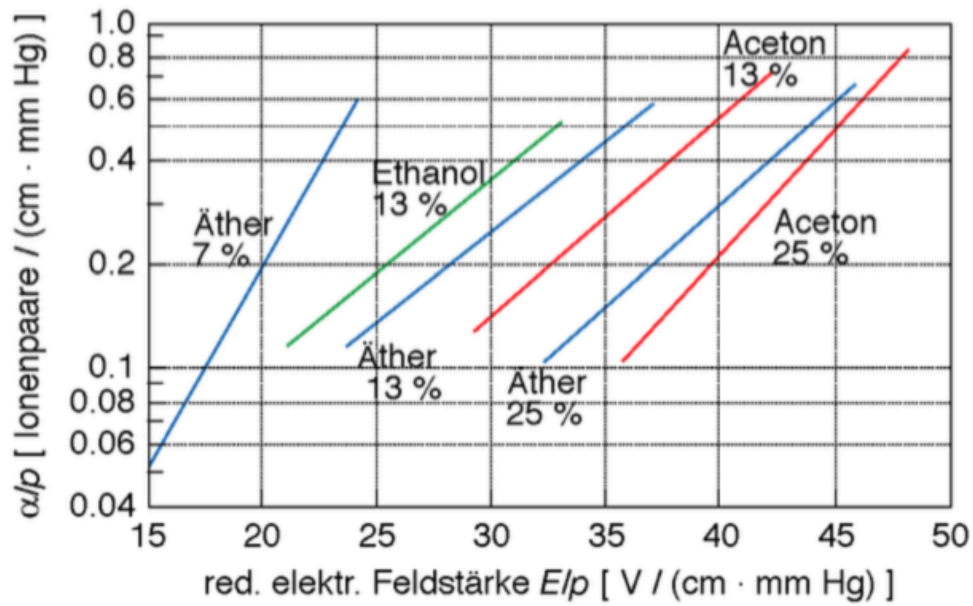
- α = number of e- an avalanche e- creates per cm.
- Adding CO2 reduces the gain.
- Calculated by Magboltz for Ar/CO2 at 3 bar.



First Townsend coefficient – Examples

- First Townsend coefficient as function of the reduced electric field strength for various noble gases and Argon with different admixtures:

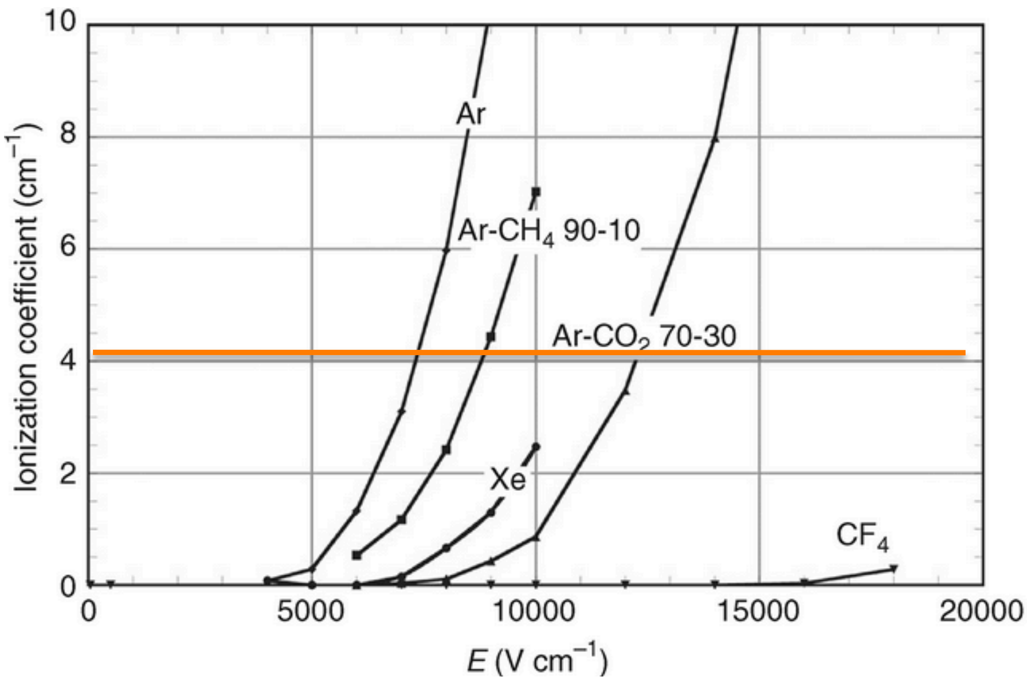




F. Sauli, *Principles of Operation of Multiwire Proportional and Drift Chambers*, CERN 77-09, 1977

The Townsend coefficient is proportional to the gas density and therefore to the pressure P.
The ratio α/P is a function of the reduced electric field

First Townsend coefficient parameterization



$$\alpha / P = A e^{-BP/E}$$

$$\alpha = k N \varepsilon$$

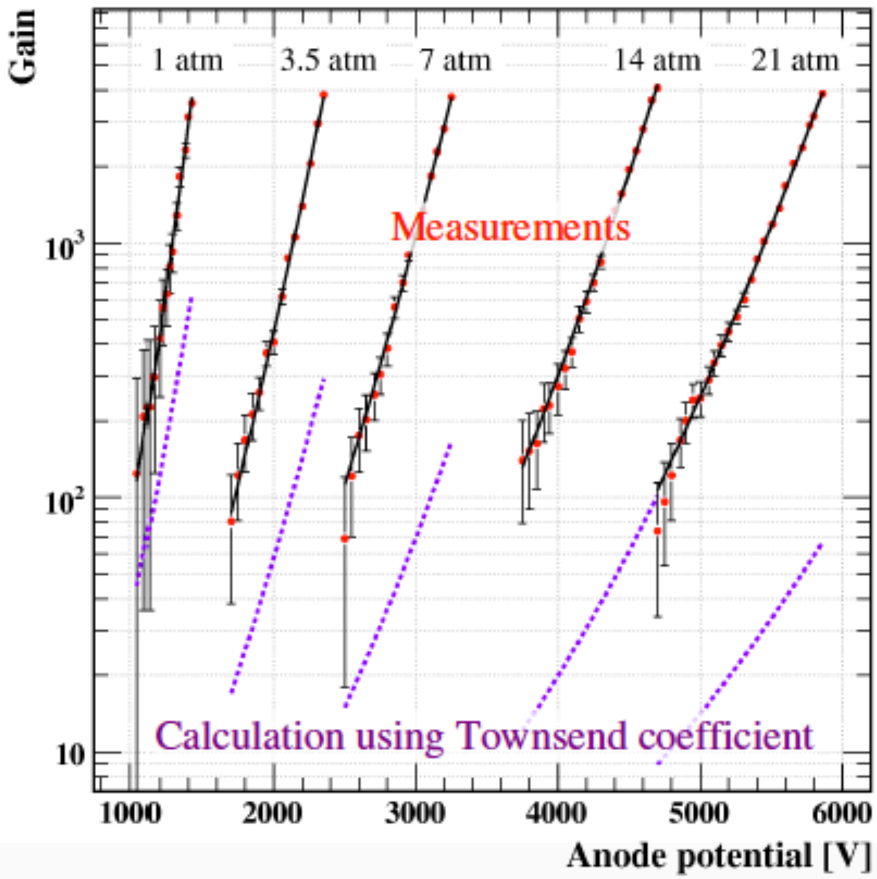
ε =average electron energy

Gas	A (cm^{-1}torr)	B (V cm^{-1}torr^{-1})	k (cm^{2}V^{-1})
He	3	34	0.11×10^{-17}
Ne	4	100	0.14×10^{-17}
Ar	14	180	1.81×10^{-17}
Xe	26	350	
CO_2	20	466	

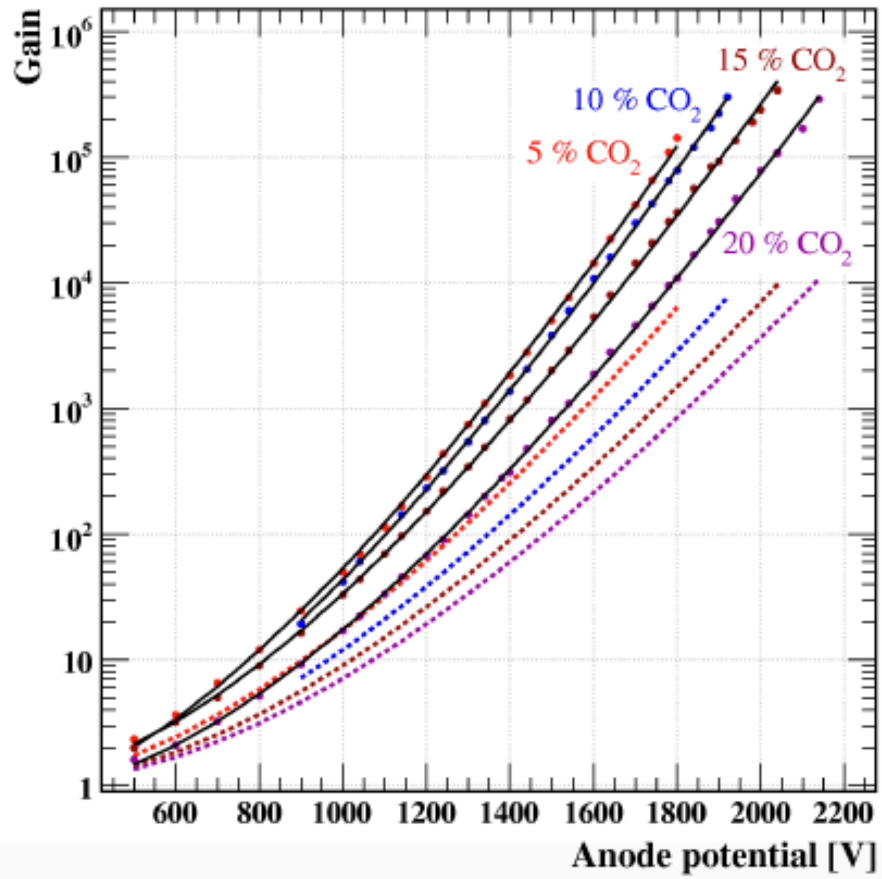
A Depends also on pressure and temperature.
Given the double exponential dependence of the gain on P,T, even a small increase in T,P could affect significantly the gain

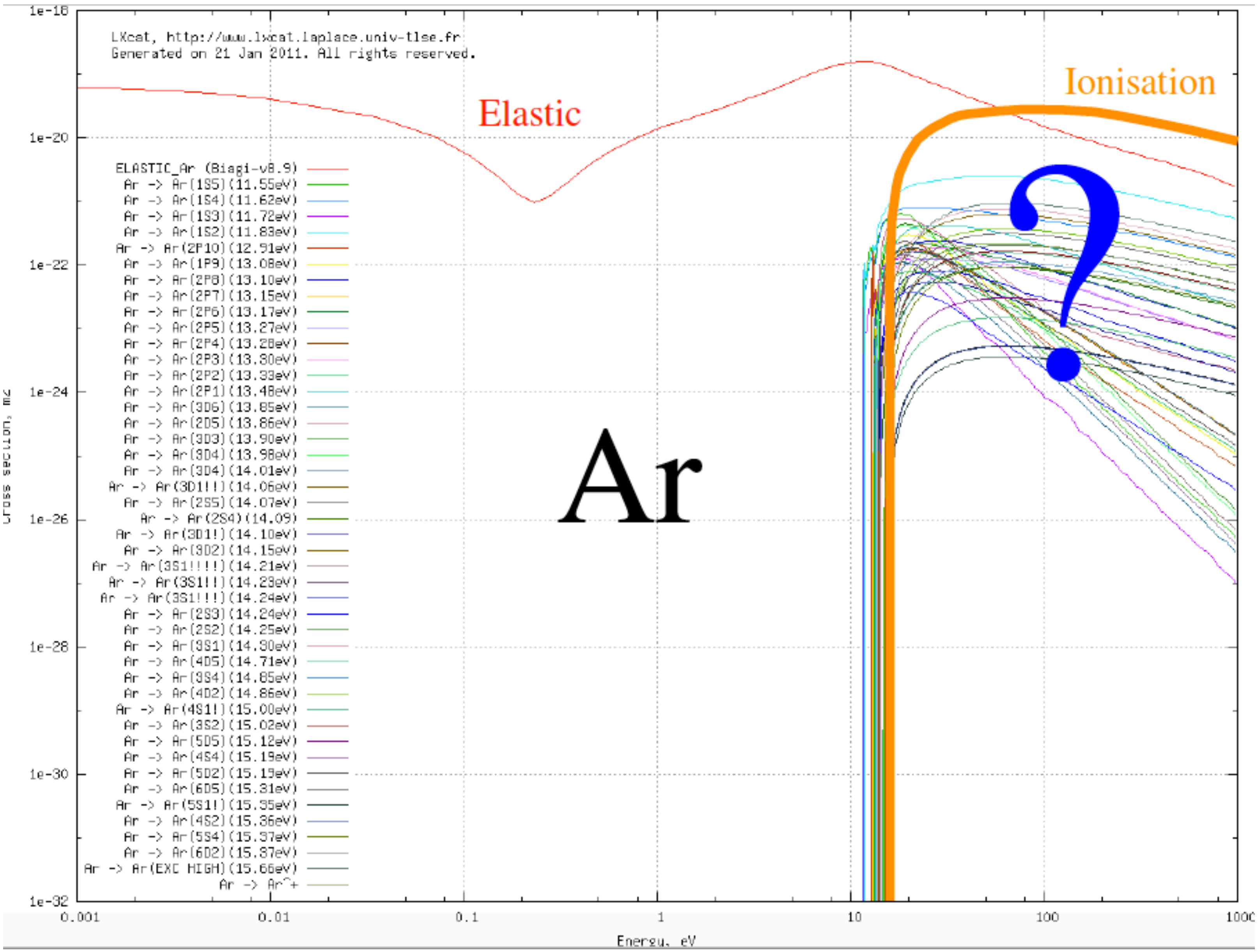
Does this reproduce the measurements ?

Ar - CH4



Ar - CO2





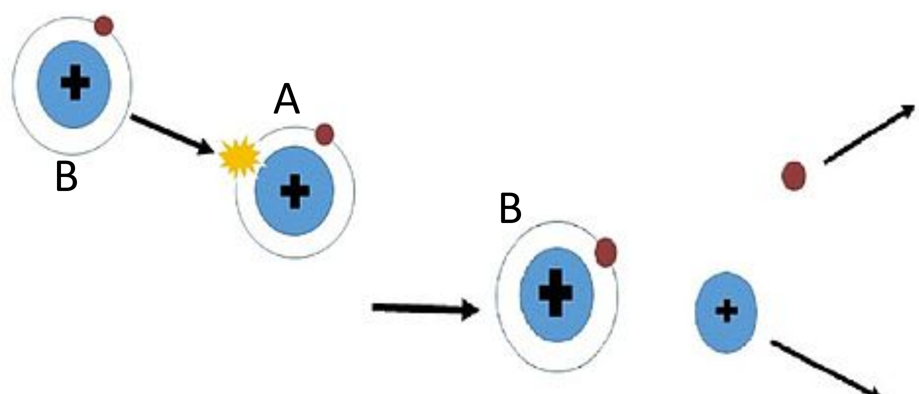
Many complex processes can occurr in Argon and other gas molecules due to the energetic structure

Amplification: Penning Effect

- The Penning effect occurs in gas mixtures, in which a **metastable excited state of one gas** component is **energetically higher than the ionization energy of the second gas component**.
  - Usually the (polyatomic) gas molecules have lower ionization potential wrt to the noble gases
- The excited gas atoms ionize the second gas through collisions → increase of the number of electron ion pairs.
- Penning gas mixtures consist typically of a noble gas (in most cases Ar) and a low concentration admixture of a molecular gas (lowe ionizing potential).
- Example:
  - **Argon, metastable states at 11.55, 13, 14 eV with admixture of Isobutane *I*=10.7 eV**
- However, excitation of vibration and rotation states in molecular gases reduces energy available for ionization.

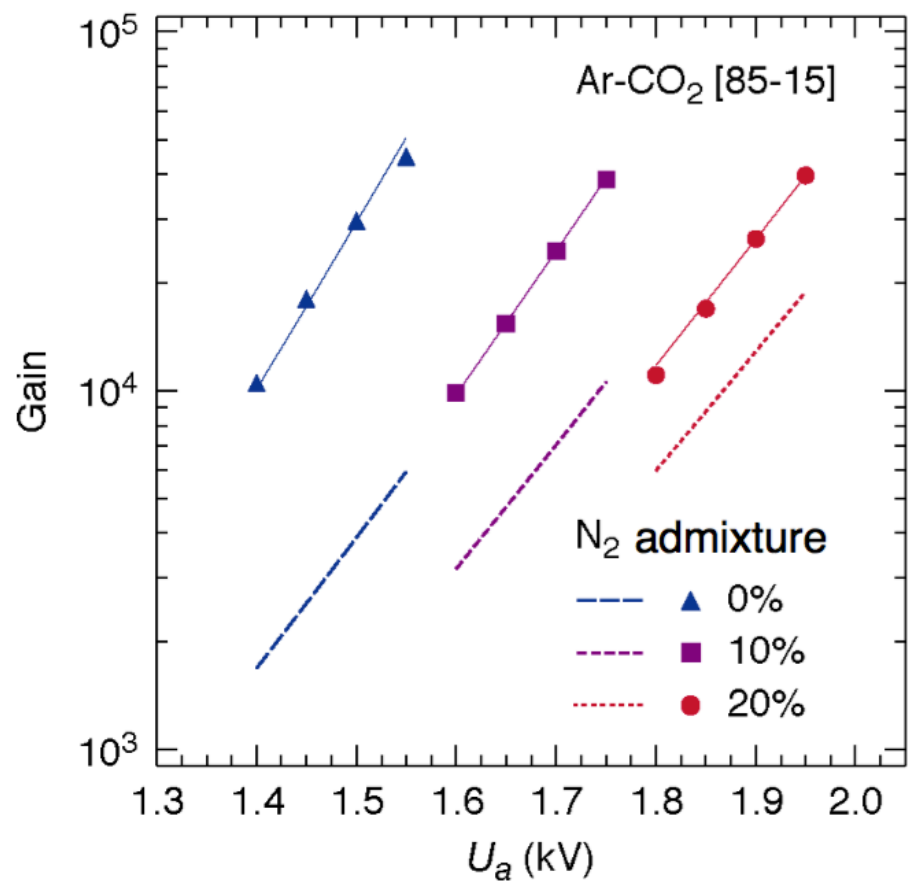
$$A^*+B \rightarrow A+B^+ + e$$

Energy $A^* >> I_B$



Amplification: Penning Effect

- Amplification in a mixture of Ar (85%) and CO2 (15%) with different admixtures of N2:



A. Andronic et al., Nucl. Instr. Meth. A **523** , 302 (2004)

Penning effect:
$Ar^* + CO_2 \rightarrow Ar + CO_2^+ + e^-$

Points: measurements
Dashed lines: simulation without Penning effect
Continuous lines: simulation with Penning effect

Some detailed examples of Penning Effect

Ar* $3p^5 4s$ can transfer to iC_4H_{10}, C_3H_8 and C_2H_6;

- two 4s are metastable, the two others live 2.6 ns and 8.6 ns;

Ar* $3p^5 4p$ can also ionise CH4;

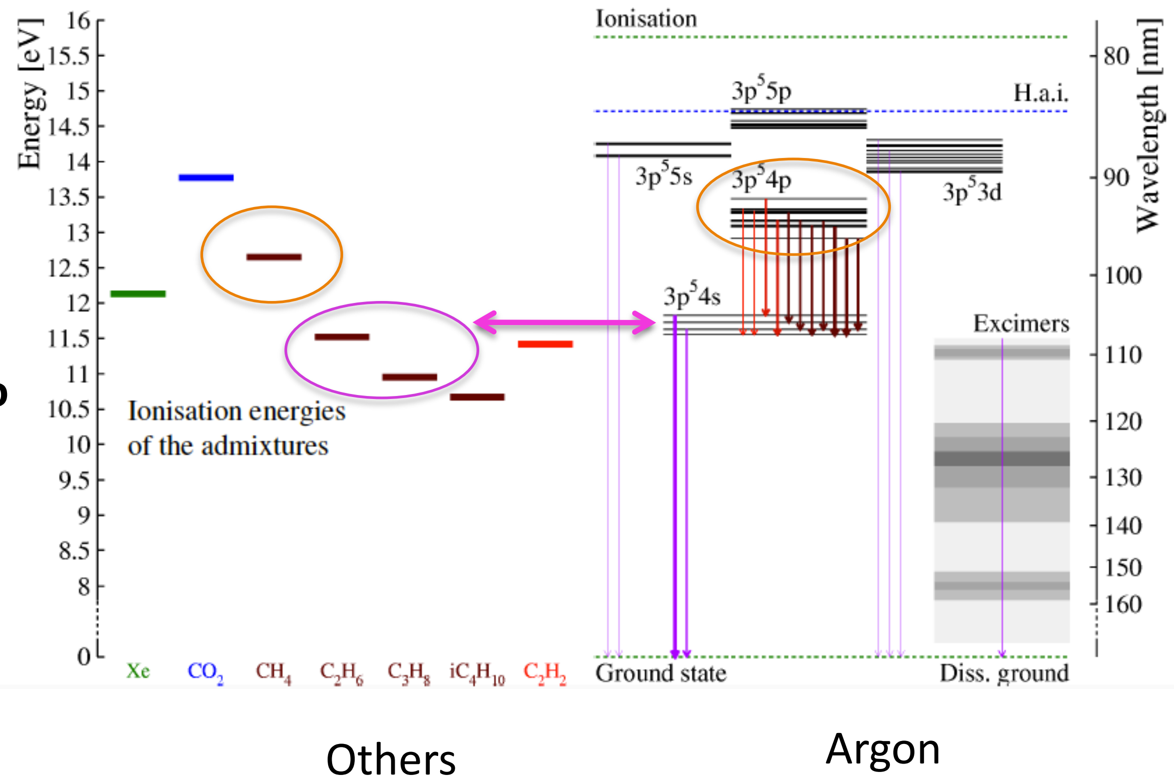
- 4p decays to 4s with a lifetime of 20-40 ns;

Ar* $3p^5 3d$ can in addition transfer to CO2;

- radiative 3d decays take ~3.5 ns, the others ~50 ns.

- For comparison, collision frequencies of Ar* in pure quencher are ~100 ps.

Level diagram argon and admixtures



Cross-check: determination of the Penning parameter

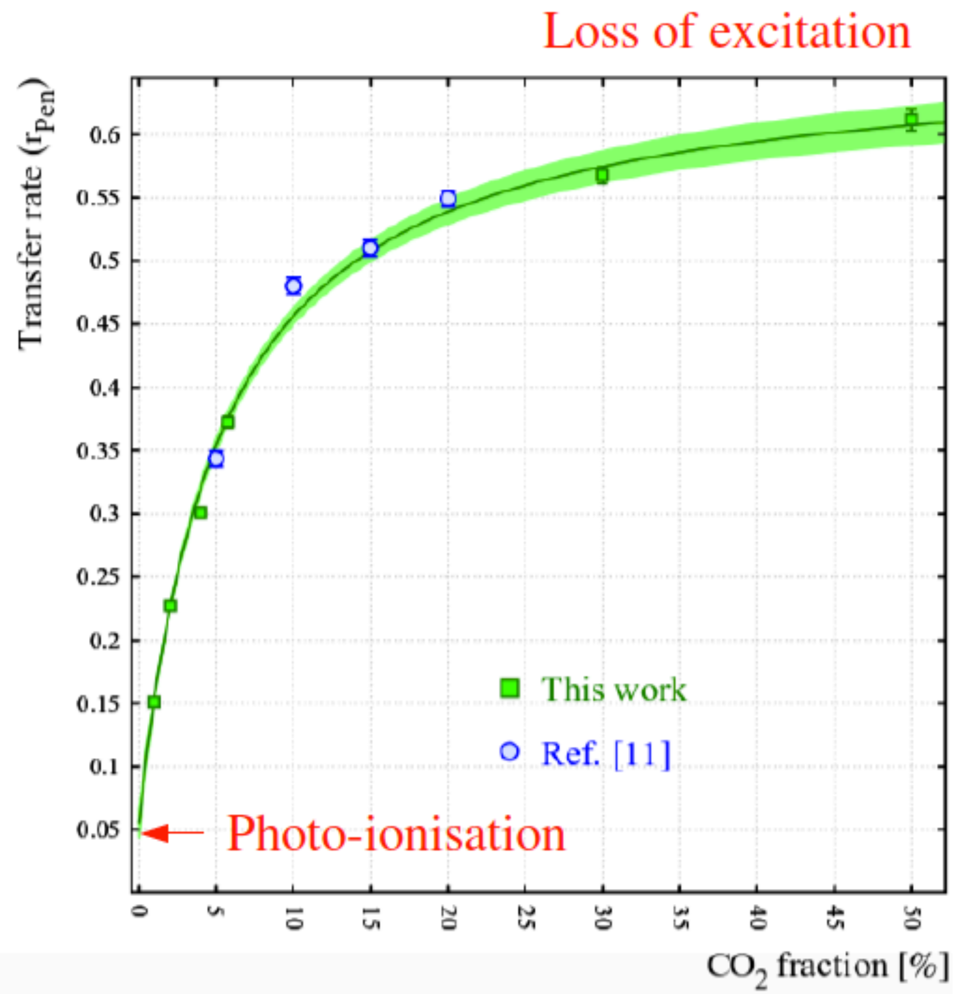
- The Penning transfer rate r(p,c) is measured by finding,in experimental data, the **fraction of excitations to be added to α**:

$$G = \exp \int \alpha \left(1 + r(p,c) \frac{\nu_{exc}}{\nu_{ion}} \right)$$

- The model parameters may be found by fitting:

$$r(p,c) = \frac{p\,c\, f_{B^+}/\tau_{AB} + p(1-c) f_{A^+}/\tau_{AA} + f_{rad}/\tau_{A^*}}{p\,c\,(f_{B^+} + f_B)/\tau_{AB} + p(1-c)(f_{A^+} + f_{\bar{A}})/\tau_{AA} + 1/\tau_{A^*}}$$

$A^* \rightarrow B^+$ $\quad$ $A^* \rightarrow A^+$ $\quad$ $A^* \rightarrow A\,\gamma$



Penning parameter fits with data from Tadeusz Kowalski et al. 1992 and 2013.
At p = 1070 hPa.

Second Townsend coefficient γ

- Gas atoms which are **excited generate UV photons**: they will induce photoelectric effect in the gas and in walls → contribute to the avalanche
  - **Loose of proportionality in the energy deposit**→this effect should be avoided
- Probability of an electron to produce a photoelectron is called the second Townsend coefficient γ

$$\gamma = \frac{\text{n. photo effect events}}{\text{n. avalanche electrons}}$$

- In the first generation the primary e^- are amplified to N_0A and produce γN_0A photoelectrons
- these are amplified in the second generation to $(\gamma N_0 A)\cdot A = \gamma N_0 A^2$ electrons and create $\gamma \cdot (\gamma N_0 A^2)$ photoelectrons, etc.

$$N(x) = N_0 A_\gamma = N_0 A + N_0 A^2 \gamma + N_0 A^3 \gamma^2 + \ldots = N_0 A \cdot \sum_{k=0}^{\infty} (A\gamma)^k = \frac{N_0 A}{1 - \gamma A}$$

- In case of **$\gamma A \to 1$** a **continuous discharge occurs**, independent from primary ionization

- **To prevent it: add a quench-gas which absorbs UV photons, leading to excitation and radiationless transitions (e.g. CH_4, C_4H_{10}, CO_2, ...)**

Avalanche breakdown

- The multiplication factor cannot be increased limitless
- Secondary ionization processes in the gas and the extraction of the photo-electrons from the walls of the detector (produced by the photons emitted in the primary avalanche) **spread the charge over the gas volume**
- This space charge **distorts the original electric field**
- Secondary avalanche can occur in front and behind the first one and may cause the transition of the proportional avalanche to the streamer and spark breakdown, if propagated along the whole detector gap
- A phenomelogical limit is given by **$\alpha x<20$ (Raether limit) or gain $<10^8$**
  - Actually, given the statistical nature of the avalanche development, the breakdown occurs for gains$>10^6$

Recipe for the gas mixture choice

Gas	Percent volume
nitrogen	78.080000
oxygen	20.950000
argon	0.930000
water	up to 4 %
carbon dioxide	0.036000
neon	0.001800
helium	0.000500
methane	0.000170
hydrogen	0.000050
nitrous oxide	0.000030
ozone	0.000004

- **Noble gas**
  - Easily ionezable
  - Do not absorb electron produced in the ionization (no attaching)
  - No polymerization
  - No flammable nor toxic, chemically inert
  - Abundant, cheap

- **Molecules with many vibrational/rotational modes**
  - Absorbs UV photons produced by the de-excitations of the gas atoms, thus avoiding discharge
  - Thanks to the Penning effect, can increase the gain
  - Thanks to the Ramsauer effect, can increase the drift velocity
  - Problems: polymerization, dissociation, attachment, flammables

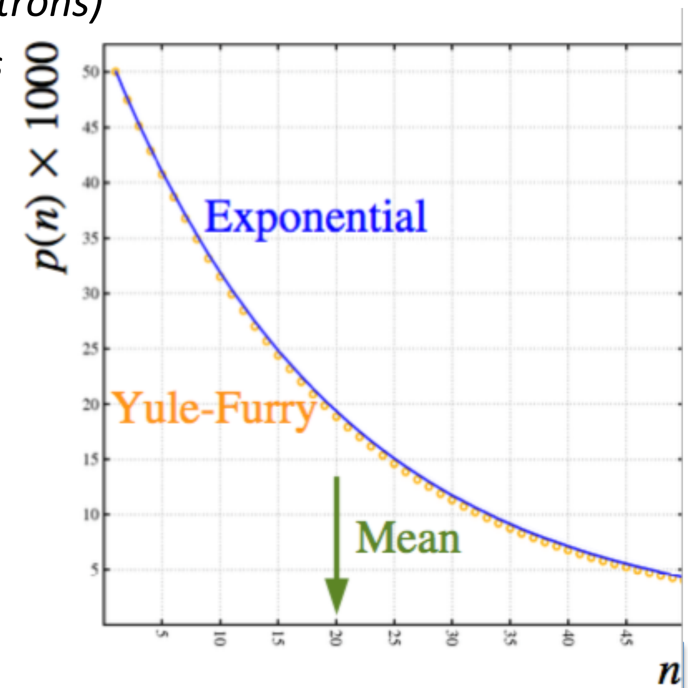
  ▪ CO2: not flammable, easily available, no polymerization

Avalanche statistics

- Statistical variation of the path covered during the ionizing collisions in the avalanche development can generate fluctuations around the average.
- Let's consider an avalanche initiated by a <u>single electron</u>
  - **n=# electrons in the avalanche**
  - **s=path covered by the single electron**
  - **1/α=mean distance between ionisations**
- P(n,s)= probability that the single electron cause n ionizing events in the path
- $P(1,s)=e^{-\alpha s}$ *Probability to not to have an avalanche*
- $P(2,s)=e^{-\alpha s}(1-e^{-\alpha s})$ *Probability to have a ionization (2 electrons)*
- $P(n,s)=e^{-\alpha s}(1-e^{-\alpha s})^{n-1}$ *Probability to have n ionization (n electrons)*
- $\langle n \rangle = 1 * e^{\alpha s}$ *mean number of electrons after a path s*

$$p(n) = \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1} \approx \frac{e^{-n/\bar{n}}}{\bar{n}-1}$$

Yule-Furry distribution

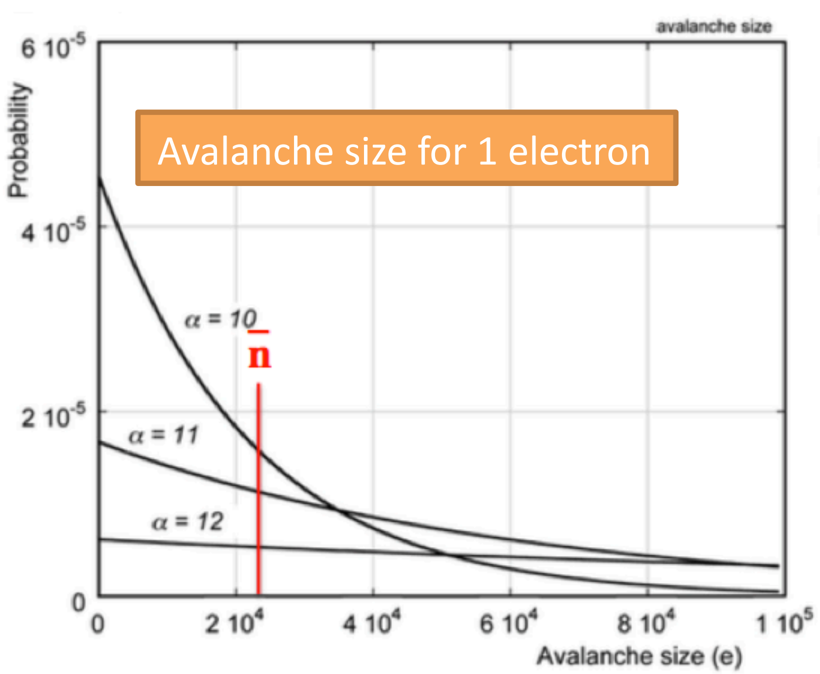


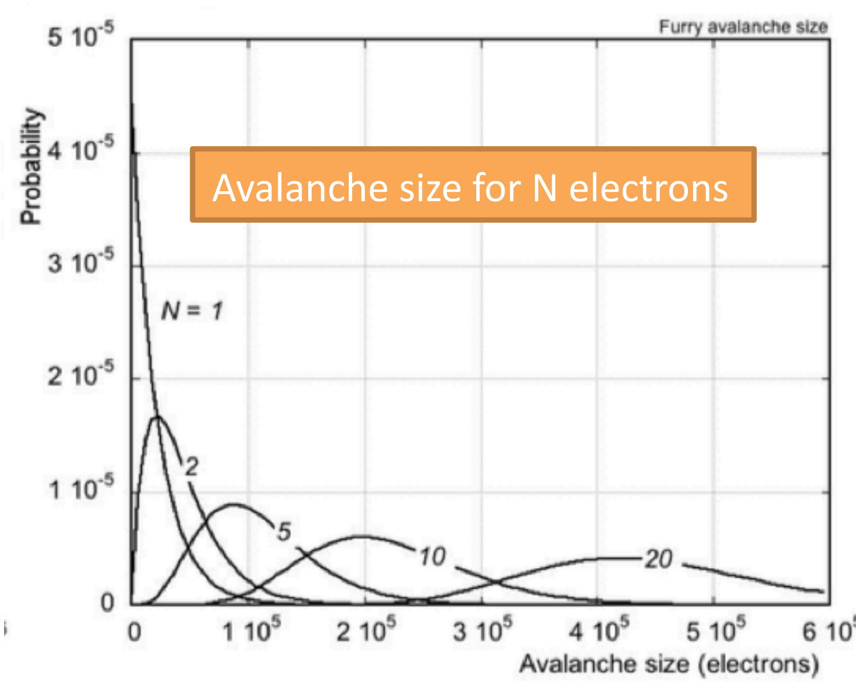
Avalanche statistics

- If the avalanche is started by N electrons, the corresponding size distribution can be obtained as a convolution of N independent exponential

$$P(n, N) = \frac{1}{\bar{n}}(\frac{n}{\bar{n}})^{N-1}\frac{e^{\frac{n}{\bar{n}}}}{(N-1)!}$$

- Signed{n} is the average avalanche size for one electron
- For N primary electrons: average avalanche size is N*Signed{n}



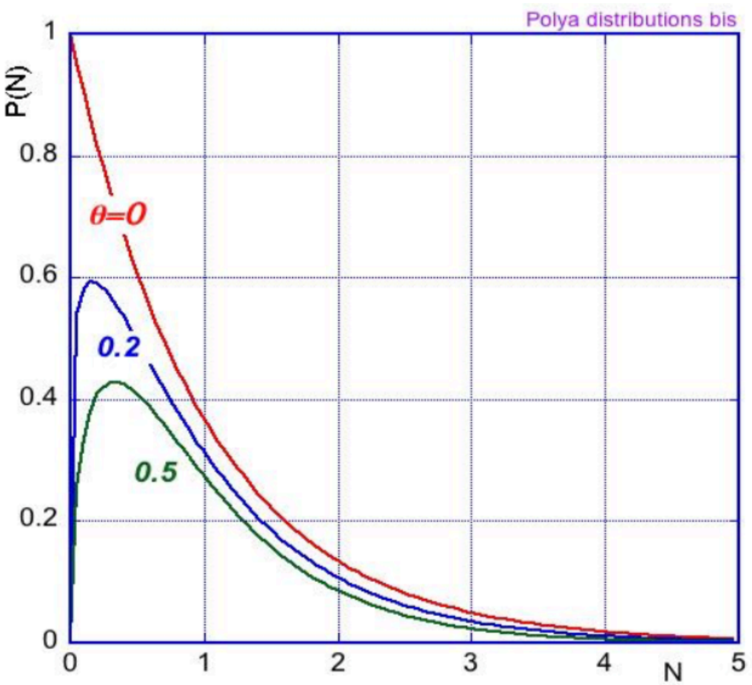


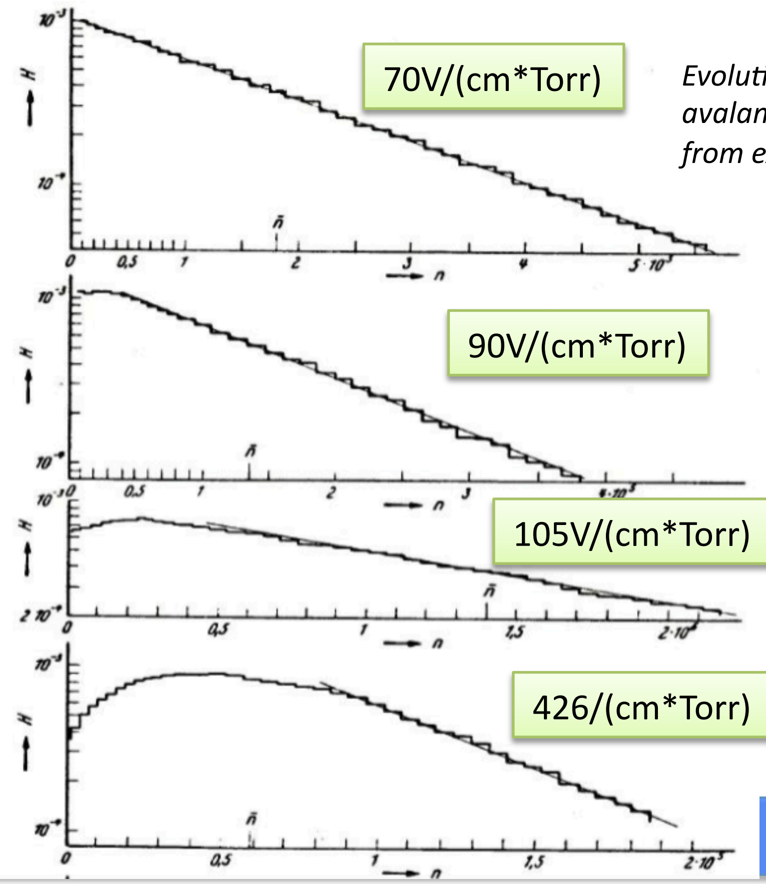
Avalanche statistics

- It has been observed that experimentally that the single electron avalanche distribution can evolve from exponential into a peaked shape for **very high value of the gain**
  - High gain→ high α →small distance between ionizing events
- The avalanche probability follows the **Polya distribution**

$$P(x,\theta) = [x(\theta+1)]^{\theta} e^{-x(\theta+1)} \qquad x = \frac{n}{\bar{n}}$$

- θ paratmeter. θ →0, P(x, θ) it's an exponential





Evolution of the avalanche size from exp to Polya

H. Sclumbohm, Zeit. Physik 151(1958)563

Operation mode

Modes of operation, depending on the strength of the electric field i.e. to the voltage applied to the electrodes

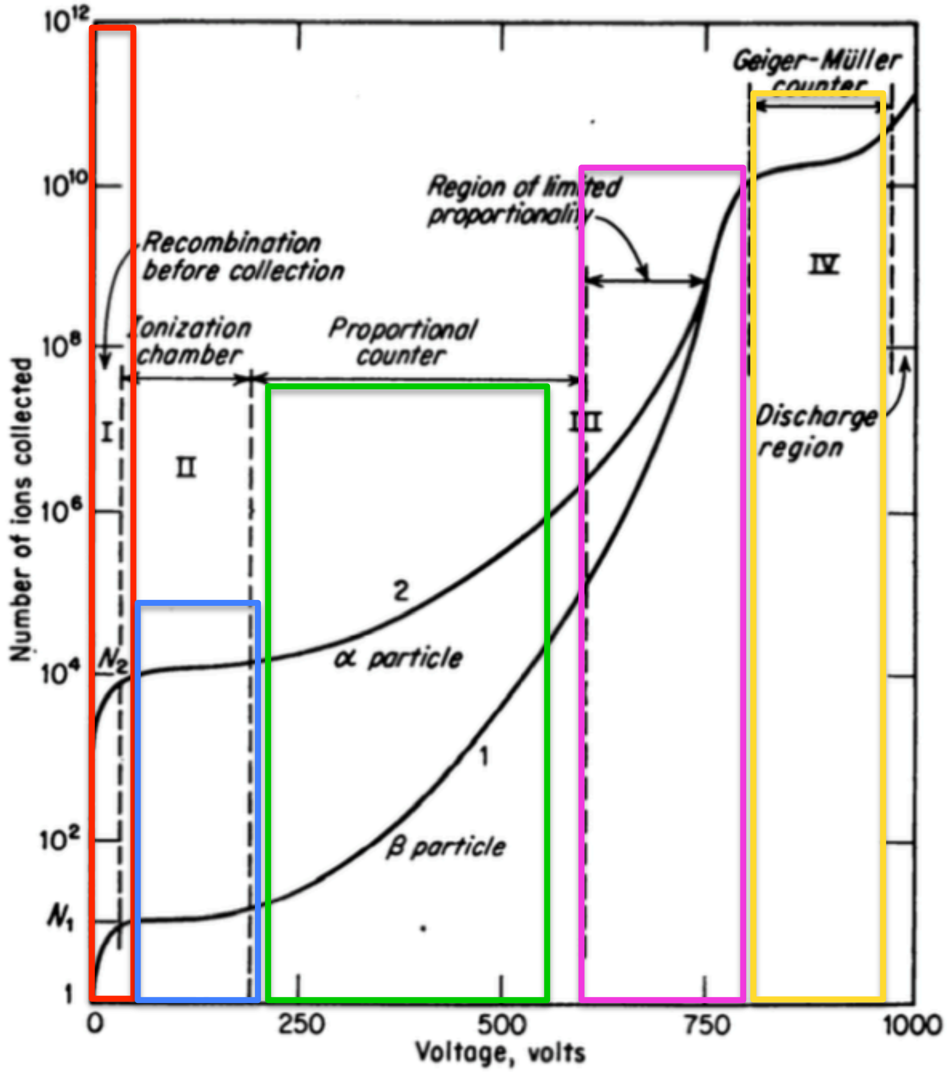


FIG. 2–2. Pulse-height versus applied-voltage curves to illustrate ionization, proportional, and Geiger-Müller regions of operation.

At 0 V the electron-ion pairs recombine

The collected charged is proportional to the energy loss of the incoming particle

The number of electron-ion pairs is flat wrt to the Electric field → working region of the ioniziation chambers

The electric field is so high that the electrons produced by the ionizing radiation gain sufficient energy to ionize nearby atoms (secondary ionization) → this can ionize other atmos →An avalanche is created by the charge multiplication

The spatial charge of the avalanche distorts the electric field → the proportionality wrt the incident radiation is lost.

Several discharge can occur in the gas (further than the one triggered by the incident radiation) because of photons emitted by de-exciting atoms that can extract electrons by the electrodes. A quenching gas can be added to drain these phenomena, so the output current has always the same amplitude.

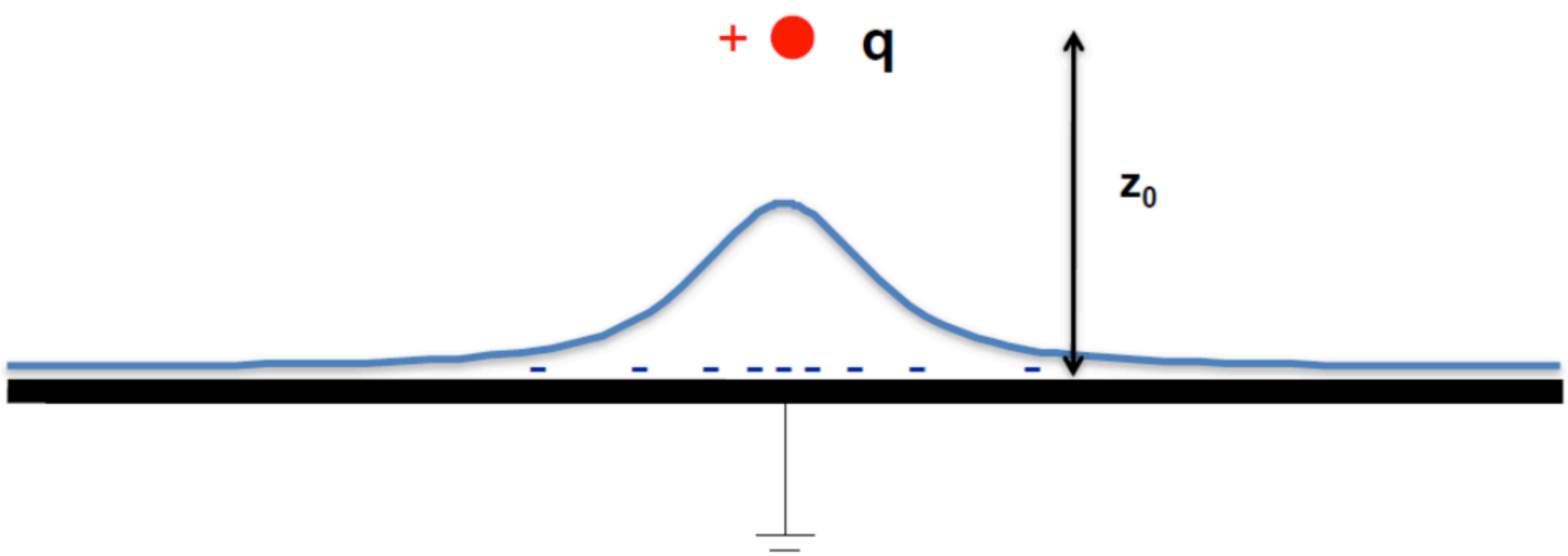
Signal

https://indico.cern.ch/event/40444/attachments/810356/1110548/detector_seminar_Werner_Riegler.pdf

Signal in gas detectors

- Remains reading the signals induced by the electrons and ions moving around in the chamber.
- The charge of the electrons and ions tries to change the voltage of the electrodes.
- The electronics compensates for this by supplying charge.

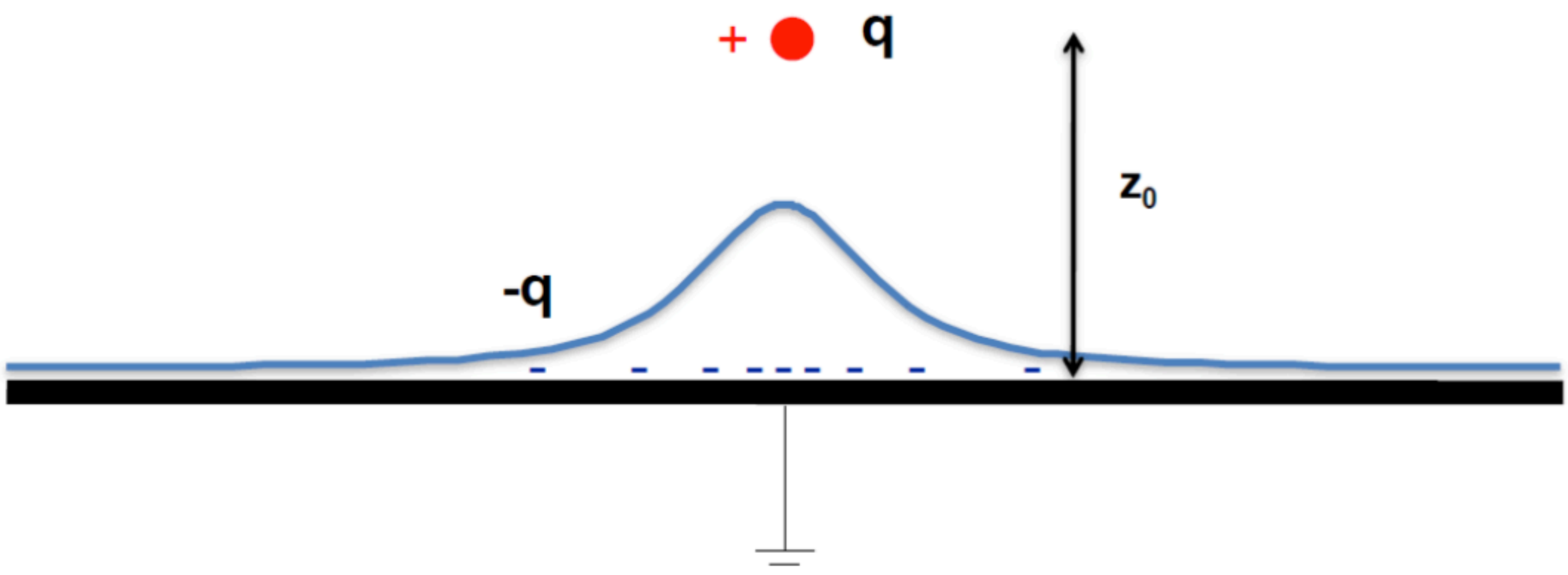
A point charge q at a distance z_0 above a grounded metal plate 'induces' a surface charge.



Charge induction

The total charge induced by a point charge q on an infinitely large grounded metal plate is equal to –q, independent of the distance of the charge from the plate.

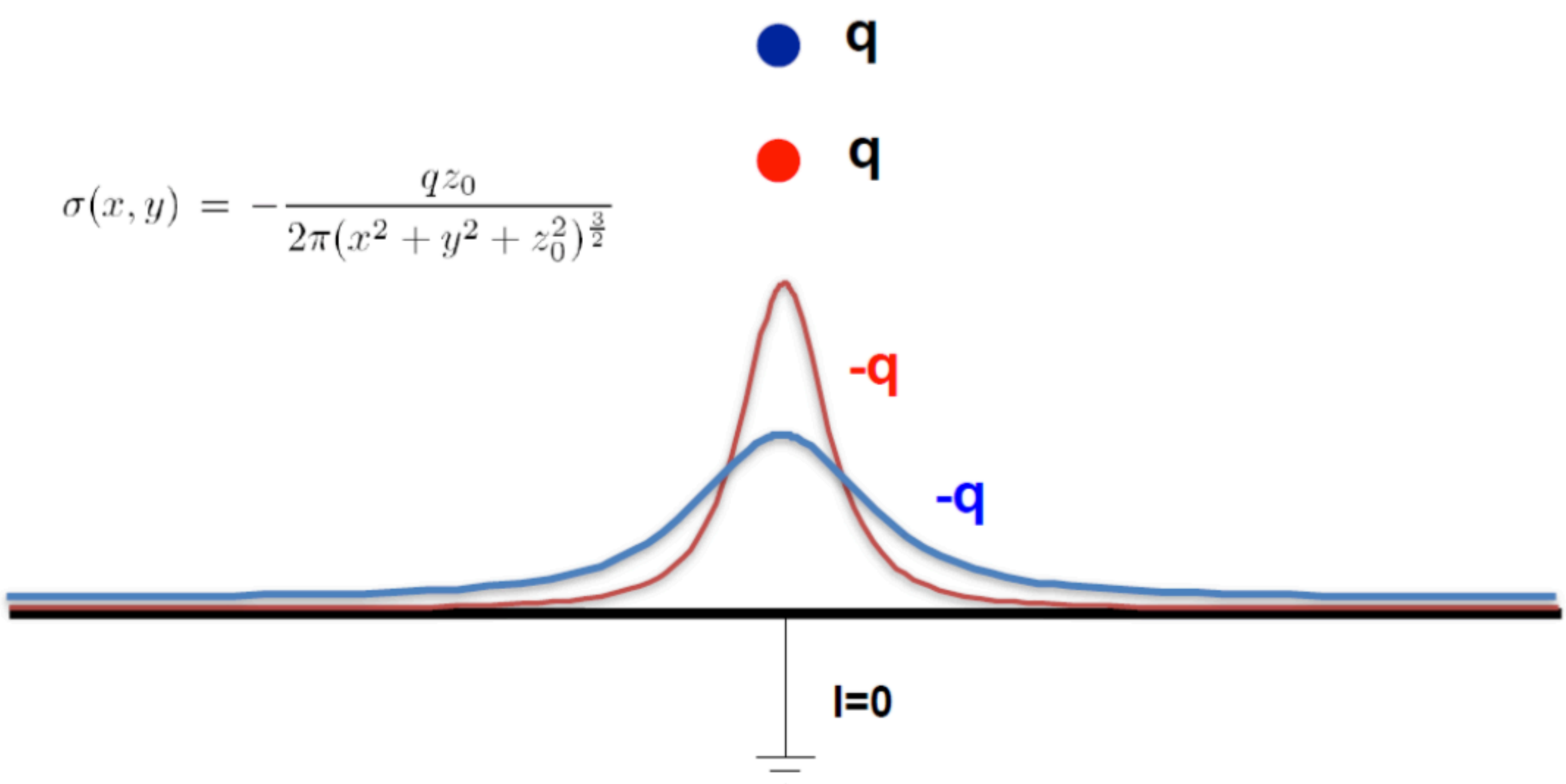
The surface charge distribution is however depending on the distance z_0 of the charge q.



Charge induction

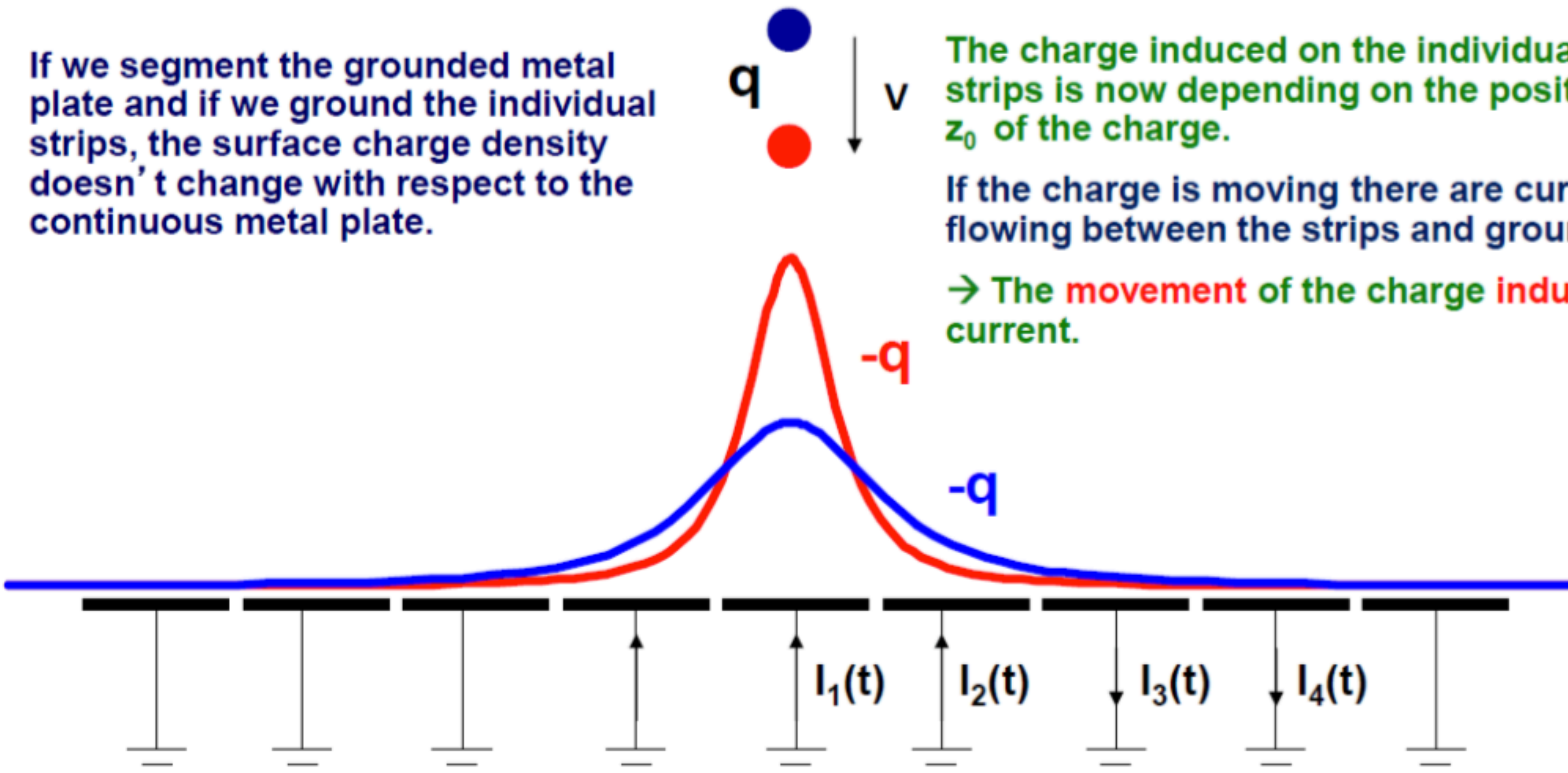
Moving the point charge closer to the metal plate, the surface charge distribution becomes more peaked, the total induced charge is however always equal to –q.

$$\sigma(x,y) = -\frac{q z_0}{2\pi(x^2+y^2+z_0^2)^{\frac{3}{2}}}$$



Charge induction

If we segment the grounded metal plate and if we ground the individual strips, the surface charge density doesn't change with respect to the continuous metal plate.



The charge induced on the individual strips is now depending on the position z_0 of the charge.

If the charge is moving there are currents flowing between the strips and ground.

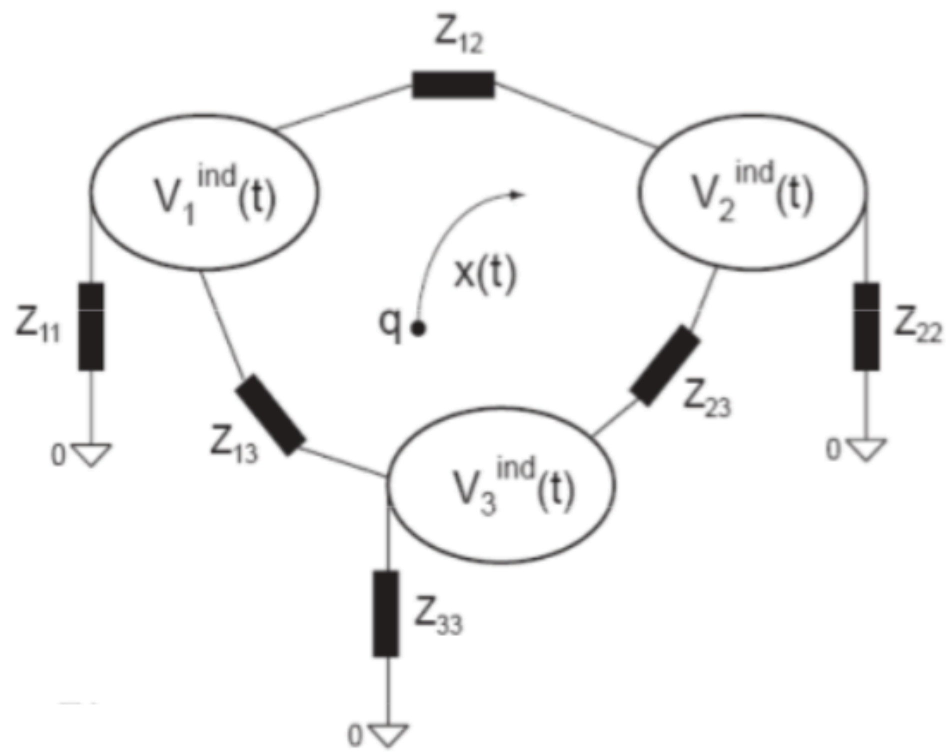
→ The movement of the charge induces a current.

$$Q_1(z_0) = \int_{-\infty}^{\infty}\int_{-w/2}^{w/2} \sigma(x,y)dxdy = -\frac{2q}{\pi}\arctan\left(\frac{w}{2z_0}\right) \qquad z_0(t) = z_0 - vt$$

$$I_1^{ind}(t) = -\frac{d}{dt}Q_1[z_0(t)] = -\frac{\partial Q_1[z_0(t)]}{\partial z_0}\frac{dz_0(t)}{dt} = \frac{4qw}{\pi[4z_0(t)^2+w^2]}v$$

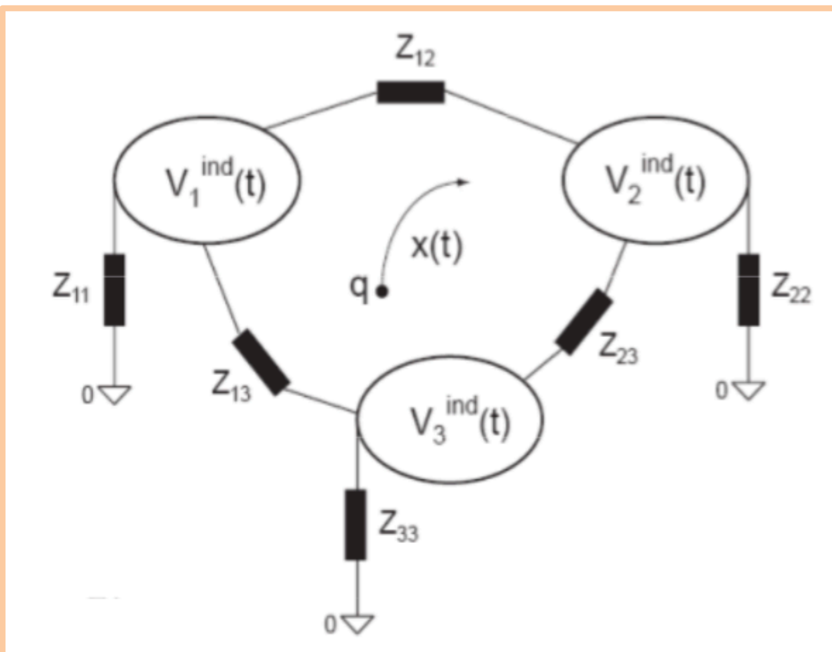
The problem

- In a real particle detector, the **electrodes** (wires, cathode strips, silicon strips, plate electrodes ...) are not grounded but they are **connected to readout electronics** and interconnected by other discrete elements.
- Question: **What are the voltages induced on metal electrodes** by a charge q moving along a trajectory x(t), in case these metal electrodes are connected by arbitrary linear impedance components ?



Formalizing the solution

1. We first calculate the currents induced on grounded electrodes.
2. Then we have to place these currents as ideal current sources on a circuit containing the discrete components and the mutual electrode capacitances



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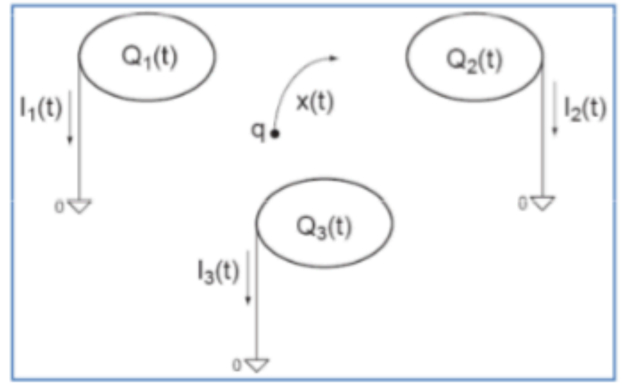

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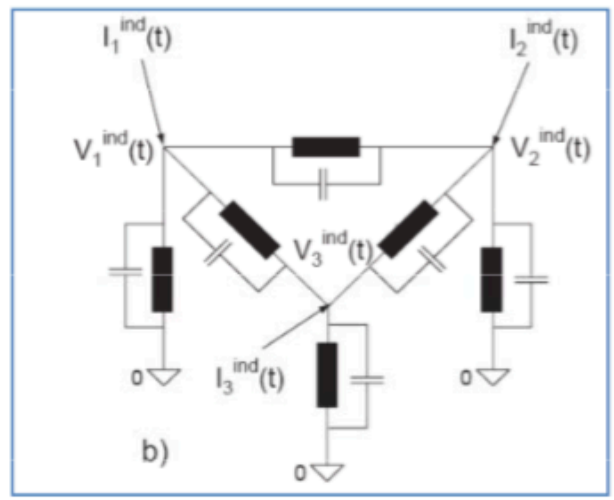
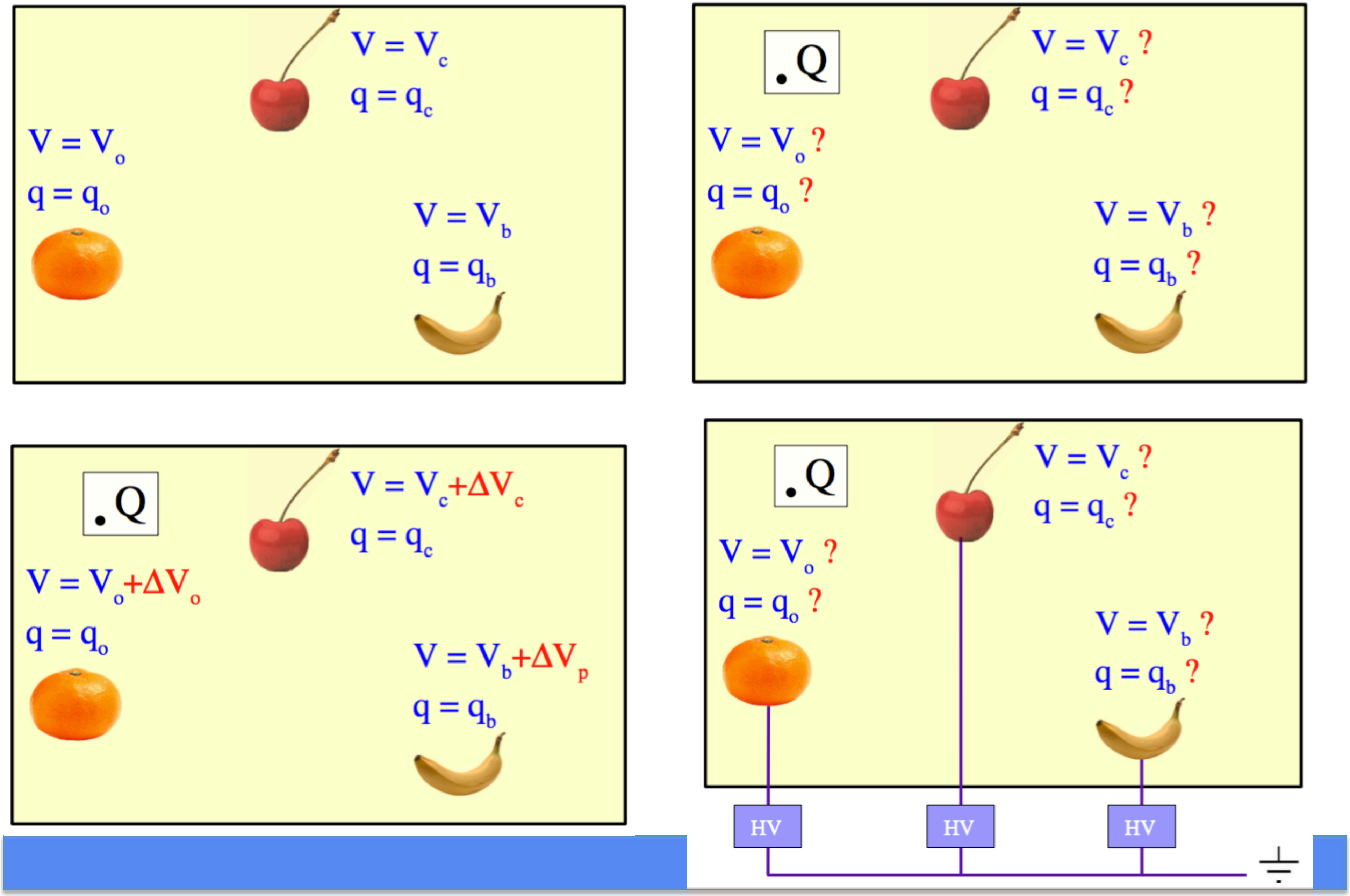
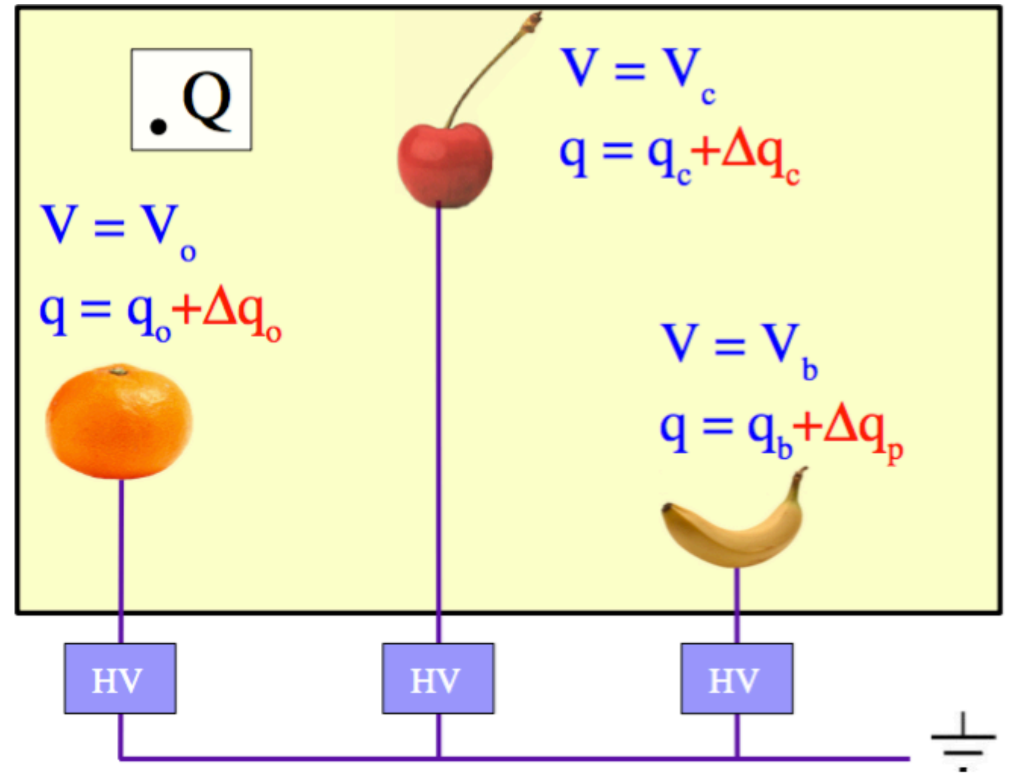


Current induction (1)



Current induction



The charge induced on the electrodes has a constraint given by

No charge creation:

$$\Delta q_o + \Delta q_c + \Delta q_p = 0$$

The Shockley-Ramo theorem

Properties of the current induced in an electrode:

- proportional to the charge Q;
- proportional to the velocity of the charge $\vec{v}$;
- dependent on the geometry. This leads to:

$$I = -Q\,\vec{v} \cdot \vec{E}_W$$

(the sign is mere convention)

- **E_w is the weighting electric field**. Each electrode has its own weighting field (depends on the geometry);
- E_w is the electric field obtained
  - **setting to 1 the electric potential of the interested electrode**
  - **by setting to 0 the electric potential to all the other electrodes**.
- This is plausible considering examples, and is proven using Green's reciprocity