Dipartimento Interateneo di Fisica "M. Merlin"
Istituto Nazionale di Fisica Nucleare

DOCTORAL COURSE XXXIII CYCLE - RESEARCH ACTIVITY REPORT I YEAR

Correlation Plenoptic Imaging and Two-Level Quantum Systems

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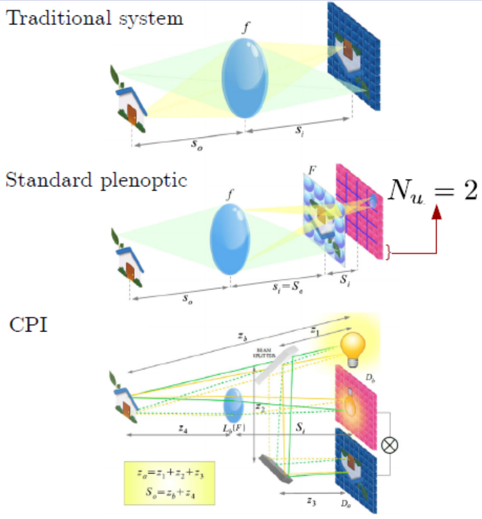
Outline

- Correlation Plenoptic Imaging (CPI)
  - signal-to-noise properties of CPI with chaotic light
  - turbulence-free imaging
- Description of Jaynes-Cummings Model
  - Extension with Broken Inversion Symmetry
  - Different approaches to investigate

Why Correlation Plenoptic Imaging CPI?



Traditional optical imaging: trade-off between resolution and depth of field

$$\Delta x = \frac{0.61\lambda}{NA} \qquad DOF = \frac{\lambda}{NA^2}$$

Standard plenoptic: only practical DOF gain

$$\Delta x = \frac{0.61\lambda}{NA} N_u \qquad DOF = \frac{\lambda}{NA^2} N_u^2$$

CPI: refocusing, 3D, no trade-off!

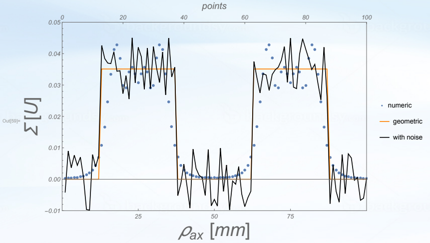
Correlation Imaging, Signal-to-noise ratio

How to characterize in correlation imaging the noise due to the refocusing?
Signal-to-noise ratio
determines the optimal number of frames to obtain fast and high-quality images.

$$\sigma(\boldsymbol{\rho}_a) = \int \Delta i_A(\alpha\boldsymbol{\rho}_a + \beta\boldsymbol{\rho}_b)\Delta i_B(\boldsymbol{\rho}_b)d^2\rho_b$$

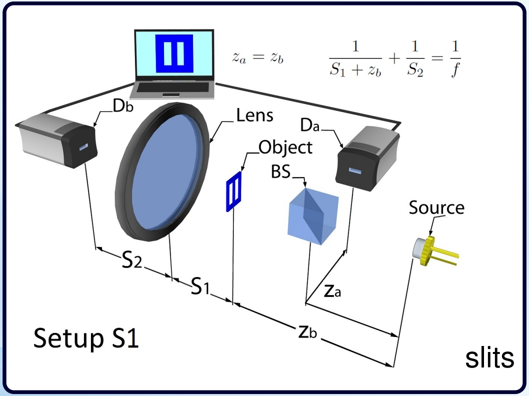
Signal (image): $\Sigma(\boldsymbol{\rho}_a) = \langle\sigma(\boldsymbol{\rho}_a)\rangle$

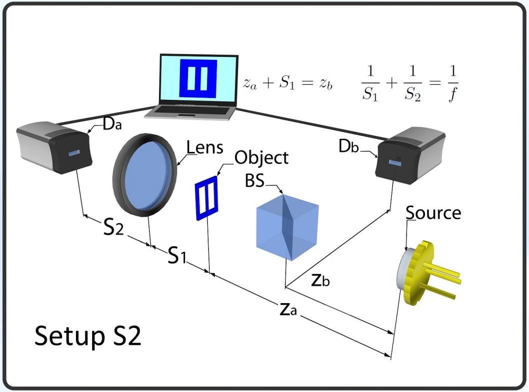
Variance: $\mathcal{N} = \langle\sigma(\boldsymbol{\rho}_a)^2\rangle - \langle\sigma(\boldsymbol{\rho}_a)\rangle^2$



α, β refocusing parameters.

Signal-to-noise ration of CPI with chaotic light





1st order image of the source
2nd order image of the object.

1st order image of the object
2nd order image of the lens.

Trade-off SNR and resolution

The SNR at the point ρ_a of the detector D_a, whit N_f number of independent frames is

$$SNR(\rho_a) = \Sigma(\rho_a)\sqrt{\frac{N_f}{\mathcal{N}}}$$

In preparation to be submitted to PRA

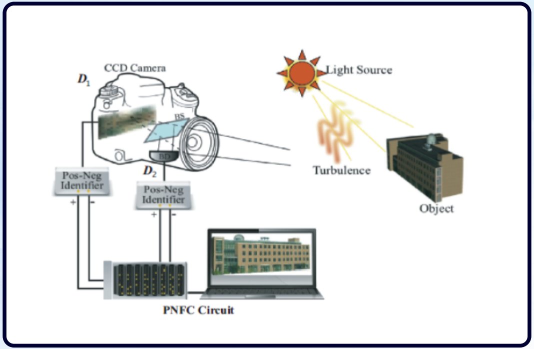
Setup 1: $\frac{SNR(\rho_a)}{\sqrt{N_f}} \sim \sqrt{\frac{a^2}{A_{\text{obj}}}}\sqrt{\frac{\Delta x^2}{A_{\text{obj}}}}|A(\rho_a)|^2$, $\Delta x = (\lambda z/a)|1 - z_b/z_a|$

Setup 2: $\frac{SNR(\rho_a)}{\sqrt{N_f}} \sim \left(\frac{S_2/S_1}{1-S_2/S_2^f}\right)^2 \sqrt{\frac{\sigma_B^2}{A_{\text{lens}}}\frac{A_{\text{lens}}}{A_{\text{obj}}}}\left|A\left(-\frac{\rho_a}{\mu}\right)\right|^2$,

$a =$ linear size of the smallest transmissive part of the object.

Turbulence-Free Imaging

Second-order imaging is robust against refraction index fluctuations which usually introduce detrimental effects in the images.



Our aim is test to robustness against turbulence of CPI protocol.

Jaynes-Cummings Model

The Jaynes-Cummings model (JCM) is a theoretical model in quantum optics for a system of a two-level atom interacting with a quantized mode of an optical cavity.

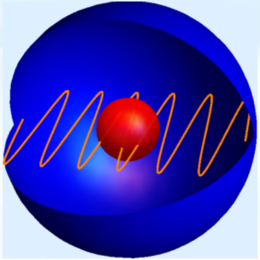
- $\sigma^+ = |e\rangle\langle g|$
- $\sigma^- = |g\rangle\langle e|$
- g_r dipole-transition coupling constant.

$$H_{JC} = H_a + H_f + H_i$$

$$H_a = \frac{\hbar\omega_a}{2}\sigma_z$$

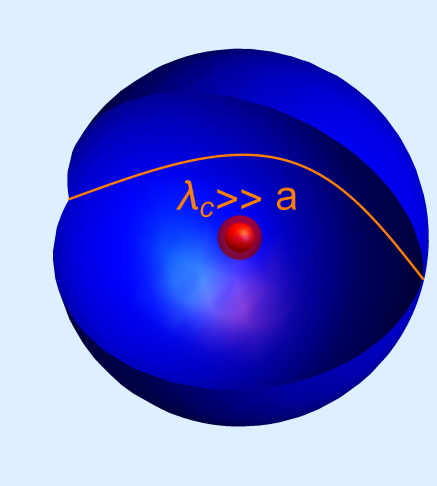
$$H_f = \hbar\omega_c a^\dagger a$$

$$H_I = g_r(\sigma^+ \otimes a + \sigma^- \otimes a^\dagger)$$

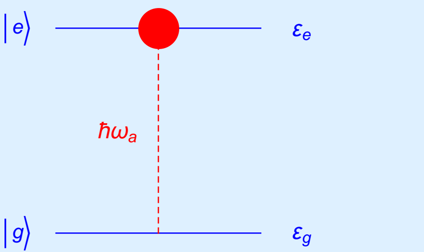


Dipole Approximation and Rotating Wave Approximation

- for real atom *a* is of the order of Bohr radius
- the dominant wavelength is in resonance with the cavity $(\omega_a \sim \omega_c)$



The Dipole Operator



The dipole operator in our basis $\{|e\rangle, |g\rangle\}$ is

$$\boldsymbol{d} = \begin{pmatrix} \boldsymbol{d}_{ee} & \boldsymbol{d}_{eg} \\ \boldsymbol{d}_{ge} & \boldsymbol{d}_{gg} \end{pmatrix} = (\boldsymbol{d}_{ee} + \boldsymbol{d}_{gg})\frac{\boldsymbol{I}}{2} + (\boldsymbol{d}_{ee} - \boldsymbol{d}_{gg})\sigma_z + \boldsymbol{d}_{eg}\sigma^+ + \boldsymbol{d}_{ge}\sigma^- .$$

The term $\boldsymbol{d}_{eg}$ is responsible for the transition between the energy levels.

What is the physical role of $\boldsymbol{d}_{ee}$?

Solvable Hamiltonian (without d_{ee})

The full Hamiltonian for symmetric system with no permanent dipole reads

$$H = \frac{\hbar\omega_a}{2}\sigma_z \otimes \boldsymbol{I}_{\mathcal{F}} + \hbar\omega_c \boldsymbol{I}_{\mathbb{C}^2} \otimes a^\dagger a + g_r(\sigma^+ \otimes a + \sigma^- \otimes a^\dagger)$$
$$= \begin{pmatrix} \hbar\omega_c(n-1) + \omega_a/2 & \sqrt{n} g_r \\ \sqrt{n} g_r & \hbar\omega_c n - \omega_a/2 \end{pmatrix}.$$

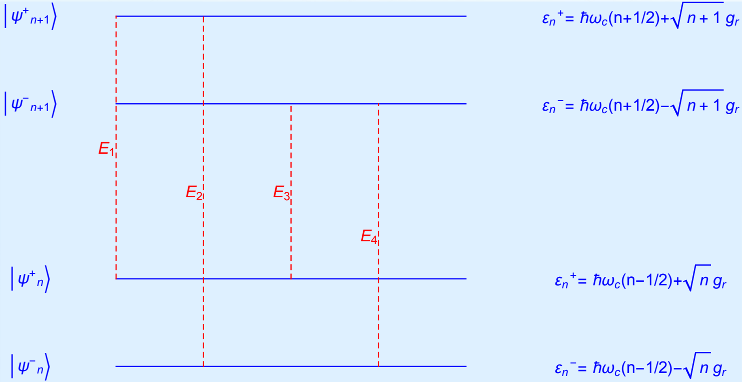
It commutes with the excitation number operator $N = a^\dagger a + (\sigma_z + \boldsymbol{I})/2$ and its eigensystem is

$$|\psi_n^{\pm(0)}\rangle = C^{\pm}_{g,n}|g,n\rangle + C^{\pm}_{e,n}|e,n-1\rangle, \quad \varepsilon_n^{\pm(0)} = \hbar\omega_c\left(n+\frac{1}{2}\right) \pm \sqrt{\hbar^2\frac{(\omega_c-\omega_a)^2}{4} + n g_r^2}$$

where $C^{\pm}_{g,n}$, $C^{\pm}_{e,n}$ are obtained by $H|\psi_n^{\pm(0)}\rangle = \varepsilon_n^{\pm(0)}|\psi_n^{\pm(0)}\rangle$.

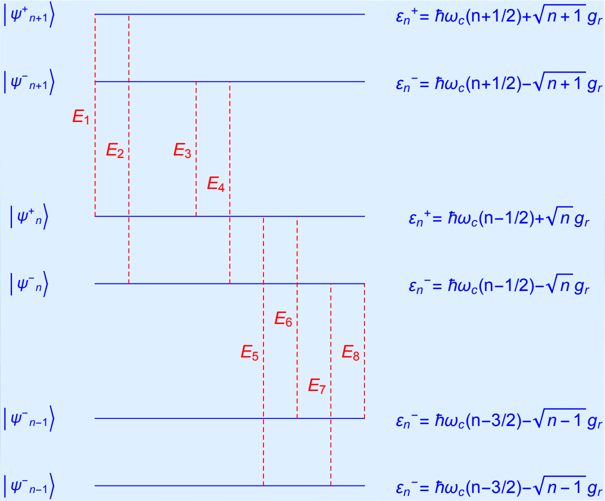
Possible Transition

Transitions for $n_f = n_i - 1$ are allowed by rotating wave approximation



$$E_{1,2} = \hbar\omega_c + g_r(\sqrt{n+1} \mp \sqrt{n}), \quad E_{3,4} = \hbar\omega_c - g_r(\sqrt{n+1} \mp \sqrt{n}).$$

Transitions at double frequencies of the cavity become opened



$$E_{1,2,3,4} = 2\hbar\omega_c \pm \hbar\frac{\Omega_R}{2}(\sqrt{n+1} \pm \sqrt{n})$$

$$E_{5,6,7} = \sqrt{n}\hbar\Omega_R$$

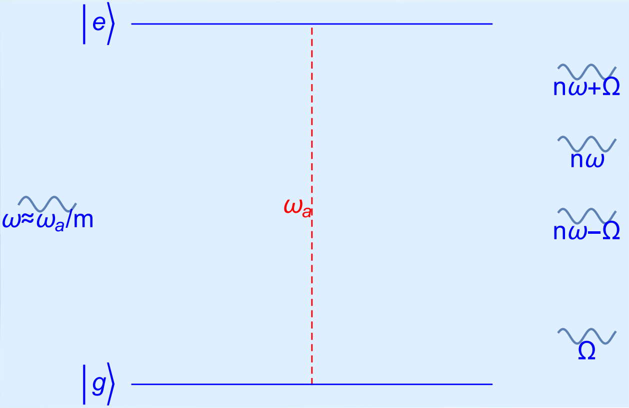
Classical Interaction

For the interaction with a classical electric field

$$H = \frac{\hbar\omega_a}{2}\sigma_z \otimes \boldsymbol{I} - \boldsymbol{E} \cdot \boldsymbol{d}\cos(\omega_c t)$$

with wavefunction $|\psi(t)\rangle = C_g(t)\,|g\rangle + C_e(t)\,|e\rangle$, where C_g, C_e are obtained by the Schrödinger equation, we can evaluate $\boldsymbol{d}(t) = \langle\psi(t)|\boldsymbol{d}|\psi(t)\rangle$.

$$\boldsymbol{d}(t) = \boldsymbol{d}_{ee}\frac{\Omega_R^2}{4\Omega^2}e^{i\Omega t} + \boldsymbol{d}_{eg}\frac{\Omega_R}{2\Omega}\sum_{n=-\infty}^{\infty} J_{m-n}(x_k)e^{in\omega t}\left[\frac{\omega_a}{\Omega} + \frac{1}{2}\left(i - \frac{\omega_a}{\Omega}\right)e^{-i\Omega t} - \frac{1}{2}\left(i + \frac{\omega_a}{\Omega}\right)e^{i\Omega t}\right]$$



- $\Omega = \sqrt{\Omega_R^2 + \Delta^2}$;
- $\Delta = \omega_c - m\omega$

NICOLAUS COPERNICUS UNIVERSITY IN TORUŃ

Exams

- ✓ Management and knowledge of European research model and promotion of research results by dr. D'Orazio;
- ✓ How to prepare a technical speech in English by prof. White;
- ✓ Programing with Python for Data Science by dr. Diacono;
- Introduction to C++ programming by dr. Cafagna;
- Linear Stability analysis by prof. Gonnella;
- ✓ Differential equations and physical phenomena by prof. Pascazio;
- ✓ States, observables and evolution by prof. Facchi;
- ✓ Atom-photon interactions by dr. Pepe.

References

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- M. D'Angelo, F. V. Pepe, A. Garuccio, and G. Scarcelli, Phys. Rev. Lett. **116**, 223602 (2016).
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acknowledgements

Thank you for your attention
any questions?

running over: “from turboless to turbulence-free”

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