PhD Project
Quantum Thermodynamics and Resource Theory

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The success of classical Thermodynamics derives from the fact that it discards the details about microscopic constituents of systems and focuses on the behaviour of some macroscopic parameters, resulting in a powerful theory for predicting the behaviour of macroscopic systems relevant in everyday life. Its strength, however, is also its weakness: as systems get ever smaller, fluctuations of quantities about their mean values become more relevant, prompting the development of a “stochastic thermodynamics”. More recently, there has been a surge of interest in exploring the quantum regime, where the origin of fluctuations is of quantum nature rather than thermal. With the advent of quantum technologies and the increasingly miniaturization of technological devices, the adaptation of Thermodynamic laws to quantum systems becomes a matter of particular importance.

Since entanglement and other quantum effects become relevant at the nanoscale, techniques of Quantum Information Theory must be used to address problems concerning Quantum Thermodynamics. The importance of Information theory for Thermodynamics can be understood also at a classical level: Landauer’s Principle [**Ben82**], stating a bound on the minimum work required to erase a bit of information, can be used to solve the famous paradox of Maxwell’s demon, assuring the validity of the second law of thermodynamics.

Issues concerning foundations of Statistical Mechanics, such as "the equal a priori probabilities postulate" can be clarified in the context of quantum thermodynamics using entanglement between the system and bath degrees of freedom. In particular, considering the Hilbert space describing the system and its environment and selecting a particular subspace with fixed energy, it has been shown [**PSW06**] that the great majority of states in this subspace give rise in the system to a canonical ensemble. This “typical” property of states has been named “Canonical Typicality”, and it can be considered in place of the postulate of equal a priori probabilities, in the sense that with a probability which approaches one in the thermodynamic limit, the state of the subsystem is indistinguishable from the state one would obtain starting from the microcanonical ensemble and tracing over the bath.

This approach does not consider the dynamical evolution of the state of a system, in particular the possibility of “untypical trajectories” having non-negligible support on states which are non-typical. For this reason one must consider the actual evolution of the system, taking into account its Hamiltonian: in this sense it is necessary to go beyond the concept of "kinematic typicality" introduced before and the "dynamical typicality" must be considered. More generally, several aspects of evolution of systems toward equilibrium are still open [**LPSW09**]. In particular it must be clarified what assumptions must be made on the Hamiltonian of the global system in order that any subsystem starting in a generic state would eventually reach the equilibrium state. One of the challenges open in the equilibration problem is to determine the equilibration times [**GHT13**], since there could be quantum systems that are going to equilibrate, but whose equilibration times are of the order of magnitude of the age of the Universe. Clearly, without bounds on the equilibration time scales, statements on equilibration become useless. It is important therefore to clarify how the equilibration times depend on the features of the Hamiltonian and the set of observable considered.

As mentioned before, in the study of thermodynamic laws on systems of small sizes the fluctuations around mean values can become important. In the framework of stochastic thermodynamics one can take care of this fluctuations viewing the basic objects of thermodynamics such as work and heat as stochastic variables and studying them through their probability distributions. In this setting, important fluctuation theorems have been obtained (see [**Sei12**] for a review) already at the classical level. At the quantum regime some important issues must be faced, for example the fact that work is not an observable and it must defined opportunely (for example with energy measurements before and after the evolution which brings the system from one state of equilibrium to the other, see [**FaGaLi17**]). These theorems are important both from a theoretical and experimental point of view. For example, in [**CRJSTB05**] the Crook Fluctuation Theorem has been used to measure RNA folding-free energies.

The most relevant results of classical Thermodynamics from a practical point of view concern bounds on the efficiency one can reach with thermal machines, therefore in the task of studying the thermodynamic laws at the quantum regime a problem that must be tackled is whether quantum effects influence the efficiency reachable on thermal machines that operate at the quantum scale. In the exploration of thermal machines techniques from dynamical control can be employed, and it can be interesting to study to what extent we can control open quantum systems to extract work using the available thermodynamic resources.

In the task of studying the laws of Thermodynamics at the quantum level, it could be useful to express the Thermodynamics as a resource theory. In general, a resource theory is defined by a restriction on the set of operations which can be effected on a system; these restrictions divide the set of the states of our system in *free states*, which can be obtained making use only of allowed operations, and *resources* which are the states that cannot be obtained using the allowed operations alone. The aim of Resource Theory is to study how these resources can be used, e.g. which state conversions are possible and which are forbidden or inhibited. This approach is clearly analogue to that of Thermodynamics, and it can be used to study its laws at the nano-scale sizes. For example, entanglement can be studied as a resource theory: two parties are allowed to do only LOCC (Local Operations and Classical Communication) on their systems; with these operations they can obtain all the separable states, but they cannot obtain entangled states: these become a resource which can be used to implement tasks which are not possible to achieve in the range of LOCC operations.

First Year Activity

During the first year of the PhD course my activity will be structured as follows:

- I will study the control of quantum systems, in particular considering both a continuous and a pulsed protocol, in continuity with my master thesis. I want also to study a generalization of the Trotter product formula which comes out by comparing the two procedures.

- Since in nature states are not generic but satisfies specific requirements, I will study the characterization of quantum Typicality when additional constraints are imposed.

- Besides the problem of typicality, also the problema of equilibration requires some physical constraints, e.g. the locality of the Hamiltonian considered. Therefore I will address the problem of equilibration taking into account additional requirements.

- Finally, I would like to study quantum fluctuation theorems in order to consider the possibility of taking advantage of states out of equilibrium as a resource, using Resource Theory.

Riferimenti bibliografici

[Ben82] Bennett C. H. *The thermodynamics of computation - A review*. Int. J. Theor. Phys. 21 905–40 (1982).

[PSW06] S. Popescu, A. J. Short and A. Winter, *Entanglement and the foundations of statistical mechanics*. Nature Phys. 2, 754–758 (2004).

[LPSW09] S. Popescu, A. J. Short and A. Winter, *Quantum mechanical evolution towards thermal equilibrium*. Physical Review E, 79.6 (2009).

[GHT13] Goldstein S, Hara T and Tasaki H, *Time scales in the approach to equilibrium of macroscopic quantum systems*. Phys. Rev. Lett., 111 (2013).

[Sei12] Seifert U. *Stochastic thermodynamics, fluctuation theorems and molecular machines.*. Reports on Progress in Physics 75.12 (2012).

[FaGaLi17] Facchi P., Garnero G., Ligabò M. *Quantum fluctuations relations*. International Journal of Geometric Methods in Modern Physics 14 (2017).

[CRJSTB05] Collin D., Ritort F., Jarzynski C., Smith S.B., Tinoco I., Bustamante C.. *Verification of the Crooks fluctuation theorem and recovery of RNA folding free energies*. Nature (2005).

[KNK15] Gelbwaser-Klimovsky, David, Wolfgang Niedenzu, and Gershon Kurizki *Chapter twelve-Thermodynamics of quantum systems under dynamical control*. Advances In Atomic, Molecular, and Optical Physics 64 329-407 (2015).

[BRLW17] M. N. Bera, A. Riera, M. Lewenstein, A. Winter, *Thermodynamics as a consequence of information conservation*. arXiv preprint arXiv:1707.01750 (2017).

[GMM09] J. Gemmer, M. Michel, G. Mahler, *Quantum Thermodynamics*. *Springer* (2009).

[GHRdRS09] Goold J., Huber M., Riera A., del Rio L., Skrzypczyk P. *The role of quantum information in thermodynamics—a topical review.* . *Journal of Physics A: Mathematical and Theoretical, 49(14), 143001* (2016).