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Quantum Thermodynamics, Control and Resource Theory

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Quantum Information Theory in Thermodynamics

Quantum Information

Resource Theory

- Foundations of Statistical Mechanics
  - Canonical Tipicality
  - Equilibration
  - Thermalization
- Quantum fluctuation theorems
- Quantum Thermal Machines

When the allowed transformations of a system are restricted, certain states of the system become useful resources.

By casting Thermodynamics in terms of a resource theory we capture its essence: not all transformations are practically realizable.

J. Gemmer, M. Michel, G. Mahler, “Quantum Thermodynamics”, *Springer* (2009)

Resource Theory

Resource theory aims at determining what is possible to do given restrictions on resources available.

- An experimenter has a limited access to a physical system
  - In which way one can influence the system?
  - What state conversions are possible?
  - In which way one can take advantage of the system?

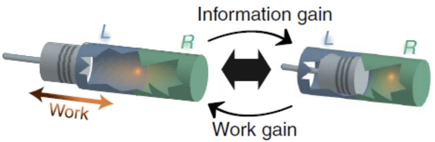
  } Thermodynamics Approach
- Resource Theory for Entanglement:
  - We are allowed to do only LOCC (local operations and classical communication).
  - Separable states can be obtained with LOCC → They are free
  - Entangled states cannot be obtained with LOCC → Entanglement becomes a resource.

M. N. Bera, A. Riera, M. Lewenstein, A. Winter, "Thermodynamics as a consequence of information conservation", *arXiv preprint arXiv:1707.01750* (2017)

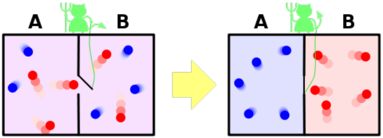
Information is physical

- **Landauer's principle**: The minimum work required to erase one bit of information is given by

$$W = k_B T \ln 2$$



- Solution of Maxwell paradox: the demon's memory must be erased $\Rightarrow$ No violation of the 2nd principle of thermodynamics.



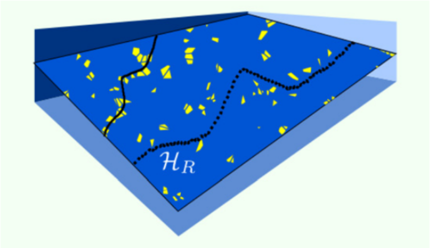
Bennett C. H., "The thermodynamics of computation - A review", *Int. J. Theor. Phys.*, 21 905–40 (1982)

Canonical tipicality

- Equal a priori postulate $\longleftrightarrow$ Canonical tipicality
  The canonical ensemble:

$$\Omega_C = \frac{e^{-\beta H_S}}{Z},$$

  with $Z = \mathrm{Tr}\, e^{-\beta H_S}$, can be obtained as a typical property of states in $\mathcal{H}_R \subset \mathcal{H}_S \otimes \mathcal{H}_B$:



$$\mathrm{Prob}\left(\rho_S \sim \Omega_C\right) \underset{d_R \to \infty}{\longrightarrow} 1$$

  where $\rho_S = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$ with $|\phi\rangle \in \mathcal{H}_R$ and $d_R = \dim\mathcal{H}_R$ ($d_R \to \infty$ in the thermodynamic limit).

- Existence of untypical trajectories.
- Equilibration of a state initially out of equilibrium.

} The Hamiltonian must be considered.

kinematic tipicality $\rightarrow$ dynamical tipicality

P. Facchi, G. Garnero, “Quantum thermodynamics and canonical typicality”, *Int. J. of Geom. Met. in Mod. Phys. World Scientific* (2017)

Equilibration and Thermalisation

- Under what circumstances do systems reach equilibrium? How much do they fluctuate?
  - Can we consider any kind of Hamiltonian?
  - Independence from the initial bath state?
  - Subsystem state independence?
- Equilibration times: Hamiltonian dependence, set of observables considered.
- Not all the states of $\mathcal{H}_R$ are reachable$\rightarrow$ Distinction between “physical” and “non-physical” states.
- Under what conditions the equilibrium state is a thermal state?

Poulin D., Qarry A., Somma R. and Verstraete F., “Quantum simulation of time-dependent Hamiltonians and the convenient illusion of Hilbert space”, *PRL* (2011)

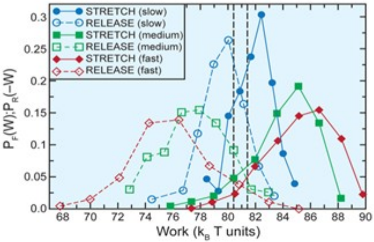
Quantum Fluctuations Theorems

Classical fluctuations theorems:

- Connect probabilities for thermodynamic quantities in forward and backward processes:

$$p_F(w) = e^{\beta(w-\Delta F)} p_B(-w)$$

- Beyond the linear response regime, systems far from equilibrium.



Collin D. et al., “Verification of the Crooks fluctuation theorem and recovery of RNA folding free energies”, *Nature*, (2005)

At the quantum level:

- Trajectories cannot be observed without perturbing the dynamics. → Work must be measured otherways.
- A large class of generalized intermediate measurements leaves the fluctuation theorems unchanged.

Quantum Thermal Machines

Issues to be adressed:

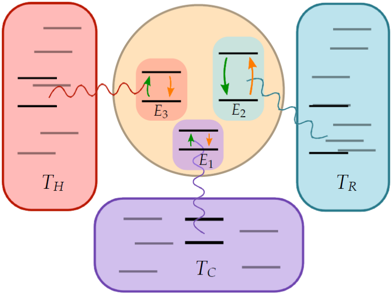
- Size limitations on the efficiency.
- Role of coherence and entanglement.

Connections with:

- Quantum Control
- Resource Theory

Potential implications in:

- Nanotechnology
- Quantum Information Processing



Schematic diagram of a three qubit refrigerator (inside the yellow circle) coupled to three thermal reservoirs.

M. Horodecki, J. Oppenheim, "Fundamental limitations for quantum and nanoscale thermodynamics", *Nature Communications* (2013).

First year activity

- Quantum Control
  - Continuous and Pulsed approach
  - Generalized Trotter Product Formula
  - Connection with thermal machines
- Quantum Typicality when the state is not generic (e.g. with requirements of symmetry or additional constraints)
- Equilibration with restricted resources (e.g. local Hamiltonians)
- Resource Theory in quantum fluctuation theorems: non equilibrium states as a resource.