DIPARTIMENTO INTERATENEO DI FISICA "M. MERLIN"

Istituto Nazionale di Fisica Nucleare

Quantum control and resource theories

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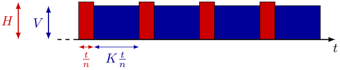
SECOND YEAR ACTIVITY

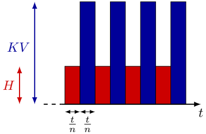
17 ottobre 2019

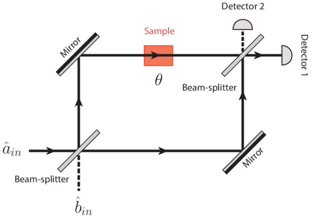
Research activity overview

Topics:

- **Quantum control**: alternating dynamics leading to dynamical superselection rules and controlled evolution
  - Generalized Pulsed evolution
  - Product formulas for alternating dynamics
- **Quantum resource theories**
  - Quantum metrology with Gaussian states
  - Entanglement resource theory







Entanglement resource theory

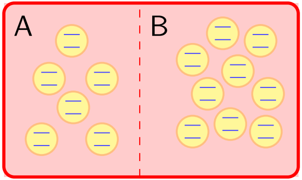
Two *distant* parties, A and B, whose systems live in $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$. Physically, the locality constraint must be imposed.

- Allowed operations (LOCC):
  - Local Unitaries
  - Local Measurements
  - Classical data communication
- Free states: separable states

$$|\psi\rangle = |\chi\rangle \otimes |\phi\rangle$$

- Resource states: entangled states

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\chi_1\rangle \otimes |\phi_1\rangle + |\chi_2\rangle \otimes |\phi_2\rangle \right)$$



Conversions between resource states:

- Deterministic conversions: $|\psi\rangle \to |\varphi\rangle$ is carried out through a protocol which never fails (if possible at all)
- Stochastic conversions: the protocol involves a quantum measurement, in which only some of the possible results lead to a successful conversion.

Conversion criteria

Given $|\psi\rangle, |\varphi\rangle \in \mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$, we say that the conversion of $|\psi\rangle$ into $|\varphi\rangle$ can happen with a success probability at most p

$$\langle\varphi| \, T(|\psi\rangle\langle\psi|) \, |\varphi\rangle \leqslant p$$

for every local operation T.

Maximal success probability

Let $|\psi\rangle \, , |\varphi\rangle \in \mathbb{C}^n \otimes \mathbb{C}^m$, with $n \leqslant m$. Defining the local density matrices

$$\psi_A = \mathrm{tr}_B(|\psi\rangle\langle\psi|), \qquad \varphi_A = \mathrm{tr}_B(|\varphi\rangle\langle\varphi|),$$

the maximal success probability in the state conversion is determined by:

$$\max_{T \in \mathrm{LOCC}} \langle\varphi| \, T(|\psi\rangle\langle\psi|) \, |\varphi\rangle = \min_{1 \leqslant k \leqslant n} \frac{\sum_{j=k}^{n} \lambda_j^{\downarrow}(\psi_A)}{\sum_{j=k}^{n} \lambda_j^{\downarrow}(\varphi_A)} \equiv \Pi(\lambda(\psi_A), \lambda(\varphi_A))$$

where $\lambda_j^{\downarrow}(\psi_A)$ denote the eigenvalues of ψ_A arranged in decreasing order.

In particular, when

$$\Pi(\lambda(\psi_A), \lambda(\varphi_A)) = \max_{T \in \mathrm{LOCC}} \langle\varphi| \, T(|\psi\rangle\langle\psi|) \, |\varphi\rangle = 1,$$

a deterministic conversion $|\psi\rangle \to |\varphi\rangle$ is possible.

High-dimensional statistics

We computed numerically

$$\Pi(\lambda(\psi_A), \lambda(\varphi_A)) = \max_{T \in \mathrm{LOCC}} \langle\varphi| \, T(|\psi\rangle\langle\psi|) \, |\varphi\rangle$$

for states $|\psi\rangle$ and $|\varphi\rangle$ sampled at random from $\mathbb{C}^n \otimes \mathbb{C}^m$ uniformly (with respect to unitary rotations), in the high-dimensional limit $n, m \to \infty$.

This task requires operations with high dimensional matrices, which can be computationally demanding.

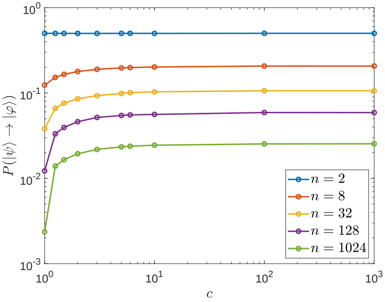
Techniques from random matrix theory allow to get the same results with a less demanding algorithm.

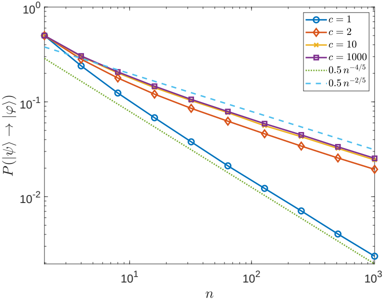
	Ordinary method	Tridiagonal method
Matrix realization	Dense structure	Sparse structure
Storage needed	$\mathcal{O}(nm)$	$\mathcal{O}(n)$
Computational Complexity	$\mathcal{O}(mn^2)$	$\mathcal{O}(n^2)$

Deterministic conversion

$$|\psi\rangle, |\varphi\rangle \in \mathbb{C}^n \otimes \mathbb{C}^m, \qquad n \leqslant m, \qquad c = m/n \geqslant 1.$$

$$P(|\psi\rangle \to |\varphi\rangle) = P\left(\max_{T \in \mathrm{LOCC}} \langle\varphi|\, T(|\psi\rangle\langle\psi|)\, |\varphi\rangle = 1\right)$$





$$\lim_{m\to\infty} P(|\psi\rangle \to |\varphi\rangle) = \kappa(n) > 0$$

$\kappa(n) > 0$ decreases in n

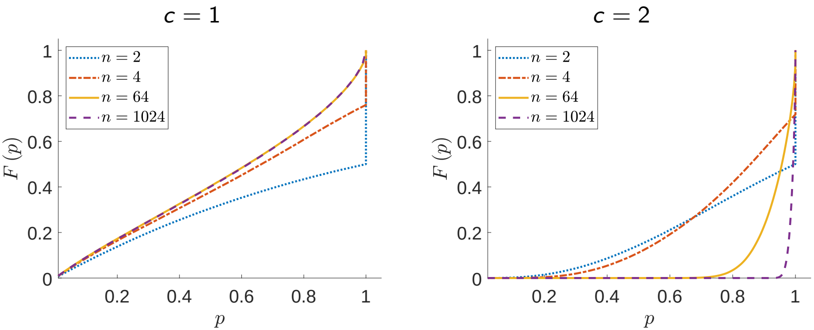
$$P(|\psi\rangle \to |\varphi\rangle) \simeq \frac{b}{n^\theta}$$

for $n, m \to \infty$ with $c = m/n$ fixed

Maximal success probability distribution

$$|\psi\rangle, |\varphi\rangle \in \mathbb{C}^n \otimes \mathbb{C}^m, \qquad n \leqslant m, \qquad c = m/n \geqslant 1.$$

$$F(p) = P\left(\max_{T \in \mathrm{LOCC}} \langle\varphi| T(|\psi\rangle\langle\psi|) |\varphi\rangle \leqslant p\right)$$



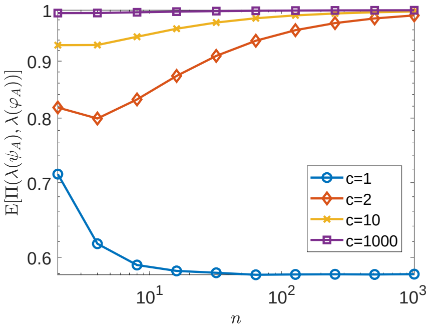
The corresponding density function $f(p) = F'(p)$ contains a continuous part and a singular one:

$$f(p) = f_{\mathrm{cont}}(p) + \kappa\delta(p-1)$$

Average success probability

$$|\psi\rangle, |\varphi\rangle \in \mathbb{C}^n \otimes \mathbb{C}^m, \qquad n \leqslant m, \qquad c = m/n \geqslant 1.$$

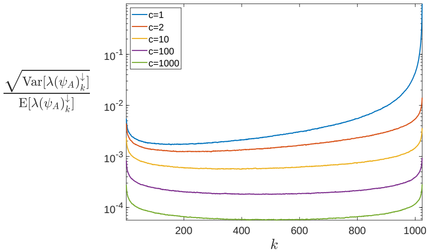
$$\mathbb{E}\left[\max_{T\in\text{LOCC}} \langle\varphi| T(|\psi\rangle\langle\psi|) |\varphi\rangle\right] = \mathbb{E}\left[\Pi(\lambda(\psi_A), \lambda(\varphi_A))\right]$$



- $c = 1$: the average success probability always decreases in n
- $c > 1$: the average success probability approaches one for large n.

Connection with minimum eigenvalue statistics

We found a connection with the fluctuations of the minimum eigenvalue of an ensemble of random matrices (here $n = 1024$):



Using techniques from Random Matrix theory we find precisely (analytically):

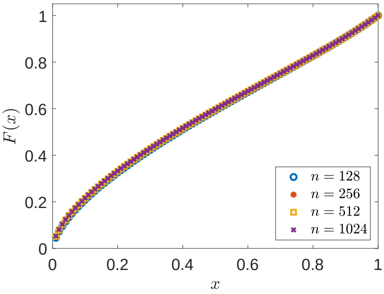
$$\frac{\sqrt{\mathrm{Var}[\lambda(\psi_A)^{\downarrow}_n]}}{\mathbb{E}[\lambda(\psi_A)^{\downarrow}_n]} \sim \begin{cases} 1 & \text{if } c = 1, \\ \frac{1}{c^{1/6}|1-\sqrt{c}|^{2/3}} n^{-2/3} & \text{if } c > 1. \end{cases} \tag{1}$$

which can be used to obtain an appropriate scaling showing a universal behaviour

Scaling and universal behaviour

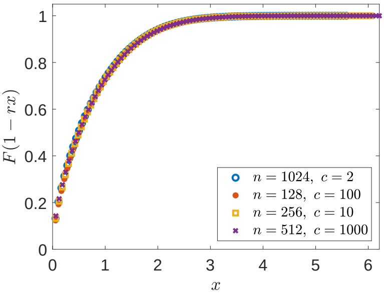
Using the connection we found with fluctuations of the smallest eigenvalue, we found also a rescaling of the variable $\max_{T\in\mathrm{LOCC}} \langle\varphi|\, T(|\psi\rangle\langle\psi|)\, |\varphi\rangle$ displaying an asymptotically universal behaviour.

$c = 1$



$$\lim_{n\to\infty} F(1-x) = 1 - H_{\mathrm{bal}}(x)$$

$c > 1$



$$\lim_{n\to\infty} F(1-rx) = 1 - H_{\mathrm{unb}}(x)$$

$$r = \frac{1}{c^{1/6}|1-\sqrt{c}|^{2/3}}\, n^{-2/3}$$

Conferences attended

- April 15$^{\text{th}}$–16$^{\text{th}}$, 2019, "Current Problems in Theoretical Physics", XXV edition, *Vietri sul Mare* (Italy);
- June 16$^{\text{th}}$–18$^{\text{th}}$ 2019, "51$^{\text{st}}$ Symposium on Mathematical Physics", *Toruń* (Poland) with **poster presentation** "Generalized product formulas and quantum control";
- September, 9$^{\text{th}}$-12$^{\text{th}}$ 2019, "12$^{\text{th}}$ Italian Quantum Information Science Conference", *Milan* (Italy); with **poster presentation** "Optimal Quantum Metrology with Squeezed states".
- September 25$^{\text{th}}$-27$^{\text{th}}$ 2019, "5$^{\text{th}}$ International Conference for Young Quantum Information Scientists", Sopot (Poland);

Publications

- D. Burgarth, P. Facchi, G. Gramegna, S. Pascazio, *Continuous and pulsed quantum control*, MDPI Proceedings. Vol. 12. No. 1. 2019. https://doi.org/10.3390/proceedings2019012015
- D. Burgarth, P. Facchi, G. Gramegna, S. Pascazio, *Generalized product formulas and quantum control*, J. Phys. A: Math. Theor. 52 435301. https://doi.org/10.1088/1751-8121/ab4403
- F.D. Cunden, P. Facchi, G. Florio, G. Gramegna, *Volume of the set of LOCC-convertible quantum states*, preprint: arXiv:1910.04646.
- G. Gramegna, D. Triggiani, P. Facchi, V. Tamma, F. Narducci, *Optimal Gaussian Metrology with squeezed states*,in preparation.

Thank you
for your attention.