UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

DIPARTIMENTO INTERATENEO DI FISICA "M. MERLIN"

INFN Istituto Nazionale di Fisica Nucleare

Waveguide QED

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Corso di Dottorato XXXII ciclo - Relazione attività di ricerca II anno

16 Ottobre 2018

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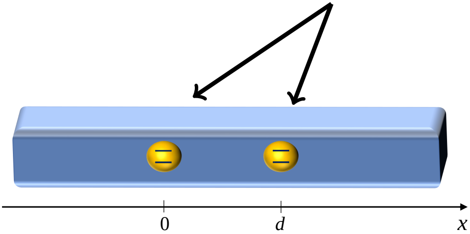
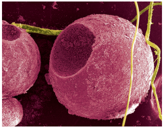
Waveguide QED

Multi-level quantum systems decay can be controlled by means of boundary conditions and dimensional reduction (e.g. embedding in a waveguide).

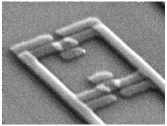
two-level atoms
A and B



Quantum Dot



SQUID



Electromagnetic field quantization:
$[a(k), a^{\dagger}(k')] = \delta(k - k')$

just photons in the lowest-cutoff energy $TE_{1,0}$ mode

States & Dynamics

The Hamiltonian is parametrized by the distance d:

$$H_d = H_0 + \lambda V_d,$$

$$H_0 = H_S + H_B = \omega_0 \big(|e_A\rangle\langle e_A| + |e_B\rangle\langle e_B| \big) + \int dk\, \omega(k)\, a^\dagger(k) a(k),$$

and the dipolar interaction is expressed with rotated couplings:

$$V_d = \int dk \sum_{s=\pm} g_s(k) b_s\, a^\dagger(k) + h.c. \quad \text{with} \quad g_s(k) = \frac{g_0(k) + s g_d(k)}{\sqrt{2}},$$

where $b_s^\dagger |g_A, g_B\rangle = \frac{b_A^\dagger + s b_B^\dagger}{\sqrt{2}} |g_A, g_B\rangle = |\Psi^s\rangle$, $b^\dagger = |e\rangle\langle g|$, $g_x(k) = \frac{e^{ikx}}{\sqrt{\omega(k)}}$.

Facchi, Pascazio, Pepe, Pomarico, in submission PRA, AX11558

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Bell basis

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The number of excitations $\mathcal{N}$ is a constant of motion, so for $\mathcal{N} = 2$:

$$|\psi\rangle = c_{AB} |e_A, e_B\rangle + \sum_{s=\pm} \int dk\ B_s(k) |\Psi^s; k\rangle + \frac{1}{\sqrt{2}} \int dk_1 dk_2\ A(k_1, k_2) |k_1, k_2\rangle$$

with $B_s \in L^2(\mathbb{R}), A \in L^2(\mathbb{R})^{\otimes 2}$ symmetric for $k_1 \leftrightarrow k_2$.

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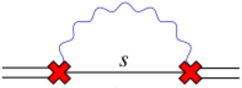
Perturbative expansion

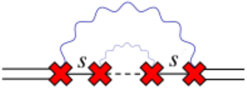
$$\omega(k) = \omega(-k)$$
$$g_s(k) = g_s^*(-k)$$

→ Parity conservation for single excitation propagators linked by a photon

Double excitation propagator lower order contributions

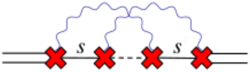


→ $O(\lambda^2)$ useful to estimate plasmon eigenstate energy



→ planar

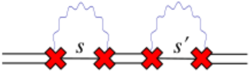
$O(\lambda^4)$



→ crossed

$$\int dq\, g_{s'}^*(q) g_s(q) F_{ss'}(z, \omega(q))$$

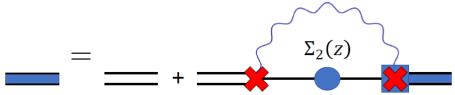
$$g_\pm^*(q) g_\mp(q) \propto \sin qd \implies s = s'$$



→ no parity constraint

Renormalized double excitation propagator

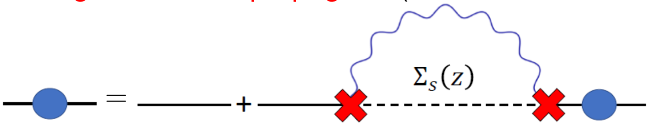
$$\langle e_A, e_B|\, \mathcal{G}(z)\, |e_A, e_B\rangle =$$



$$= \frac{1}{z - 2\omega_0 - \lambda^2 \Sigma_2(z)} = \mathcal{G}_2(z)$$

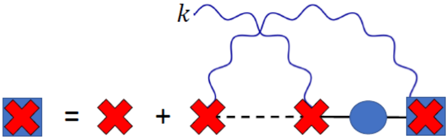
Renormalized single excitation propagator (one-excitation sector):

$$\mathcal{G}_s(z) =$$



the vertex renormalization is obtained:

$$\lambda\, s\, g_s(k)(1+X_s(z,k)) =$$

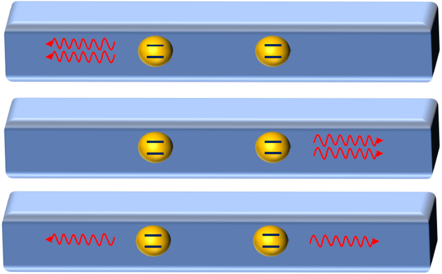


Two-photon amplitude

Asymptotic two photon state:

$$\langle k_1, k_2 | \, e^{-iHt} \, | e_A, e_B \rangle = A(k_1, k_2, t) \stackrel{t \to \infty}{\longrightarrow} \mathcal{A}(k_1, k_2) e^{-i(\omega(k_1)+\omega(k_2))t}$$

$$D_{\Leftrightarrow[\rightleftharpoons]} = \{(k_1, k_2) \in \mathbb{R}^2 | \mathrm{sgn}(k_1) = [-]\mathrm{sgn}(k_2)\}$$



$$\mathcal{P}_{\Leftrightarrow} = \frac{1}{2} \int_{D_{\Leftrightarrow}} dk_1 \, dk_2 \, |\mathcal{A}(k_1, k_2)|^2$$

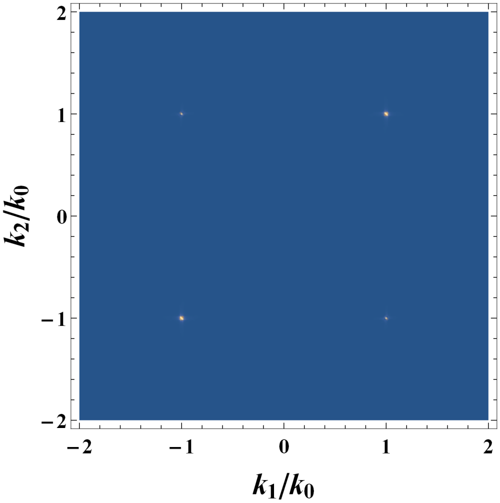
$$\mathcal{P}_{\rightleftharpoons} = \frac{1}{2} \int_{D_{\rightleftharpoons}} dk_1 \, dk_2 \, |\mathcal{A}(k_1, k_2)|^2$$

Ratio of asymptotic probabilities

$$R(\lambda)\big|_{\lambda \to 0} = \frac{P_{\Leftrightarrow}}{P_{\rightleftharpoons}} = 1 + 2\sin^2 k_0 d$$

Non-perturbative weak coupling limit

Selected parameters: $\omega_0 = 1.1M$, $\lambda = 10^{-2}M$, $k_0 d = \frac{\pi}{2}$



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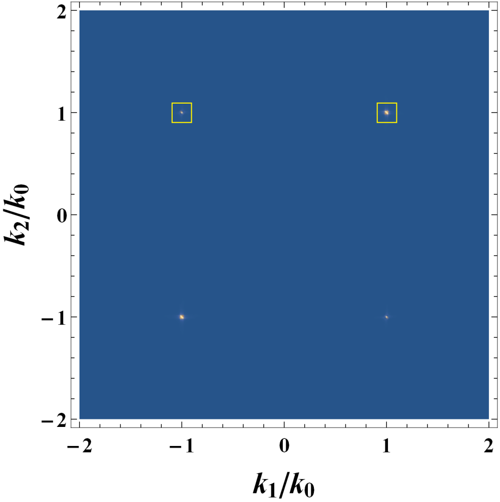

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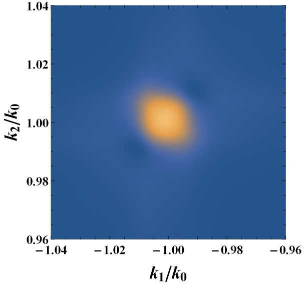
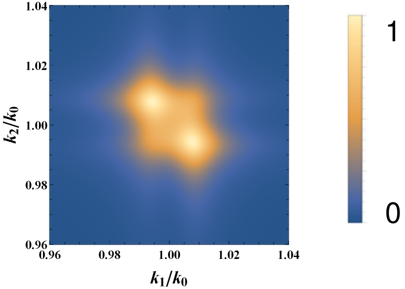
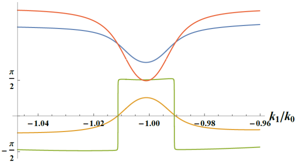


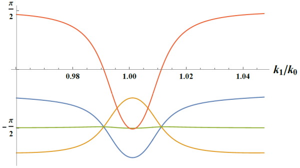
$$\frac{k_0}{M} \simeq 0.458$$

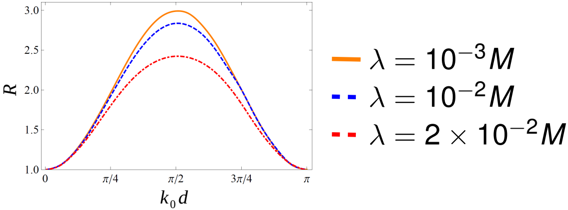


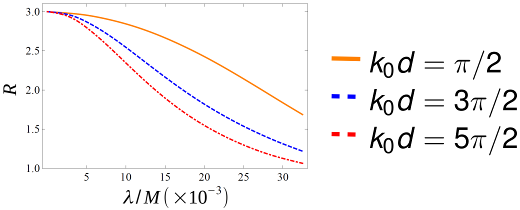
This is the parity contributions interference $\mathcal{A}(k_1, k_2) = \sum_{s=\pm} \mathcal{A}_s(k_1, k_2)$:



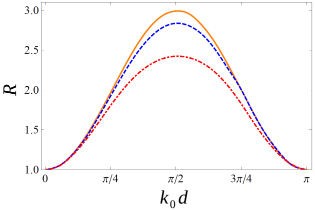
- $\phi_- = \mathrm{Arg}\{\mathcal{A}_-(k_1, k_2)\}$
- $\phi_+ = \mathrm{Arg}\{\mathcal{A}_+(k_1, k_2)\}$
- $\mathrm{Arg}\{\mathcal{A}(k_1, k_2)\}$
- $\phi_+ - \phi_-$







- $\omega_0 = 1.1M$;
- peaks in the variation of $k_0 d$ are points of the orange curve in the variation of λ;
- increased distances implement stronger coupling regimes.



- $\lambda = 10^{-3}M$
- $\lambda = 10^{-2}M$
- $\lambda = 2 \times 10^{-2}M$

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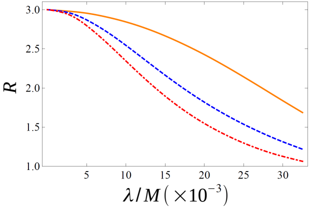
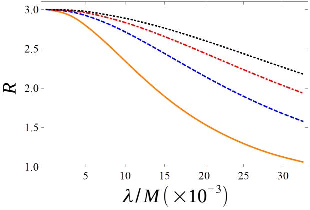


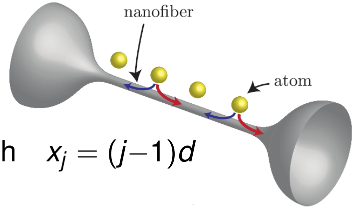
- $k_0 d = \pi/2$
- $k_0 d = 3\pi/2$
- $k_0 d = 5\pi/2$

- $k_0 d = 5\pi/2$;
- increasing values for ω_0 push forward the plasmonic bound state.

- $\omega_0 = 1.1M$
- $\omega_0 = 1.2M$
- $\omega_0 = 1.3M$
- $\omega_0 = 1.4M$



Multi-atom chain



$$H_0 = \sum_{j=1}^{N} \omega_0 b_j^\dagger b_j + \int dk\, \omega(k) a^\dagger(k) a(k),$$

$$V_d = \sum_{j=1}^{N} \int dk \left(g_{x_j}(k) b_j^\dagger a(k) + h.c. \right), \quad \text{with} \quad x_j = (j-1)d$$

Excitation amplitude array $\mathbf{c} = (c_1, \ldots, c_N)$, self-energy matrix $\Sigma(z)$

Hamiltonian eigenvalues ($E \in \mathbb{R}$): $\big((\omega_0 - E)\mathbb{1} - \Sigma(E + i0^+)\big) \cdot \mathbf{c} = 0$

Waveguide QED: $\Sigma(z) = \Sigma_{\text{pole}}(z) + \Sigma_{\text{cut}}(z)$

Facchi, Lonigro, Pascazio, Pepe, Pomarico, in preparation

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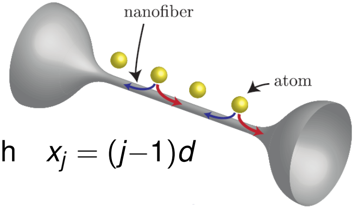

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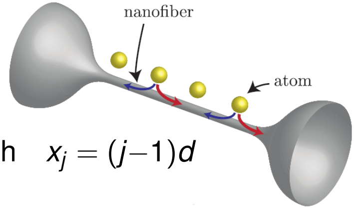

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Resonance condition $\sqrt{E^2 - M^2} d = n\,\pi$

n even: $\sum_j c_j = 0$, n odd: $\sum_j (-1)^j c_j = 0$

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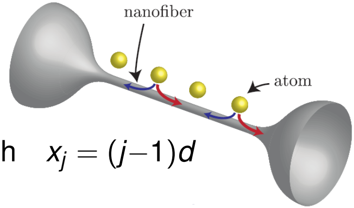

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Waveguide QED: $\Sigma(z) = \Sigma_{\text{pole}}(z) + \Sigma_{\text{cut}}(z) \longrightarrow$ parity metastability with first decaying state according to:

$$\frac{\gamma}{2} \propto \lambda^2 \frac{2(N-2)}{3N^3}$$

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Outlook

Main activities in progress for the third year are:

- variation of the initial state with two excitations;
- existance of non resonant eigenstates for the multi-atom setup;
- investigation of waveguides endowed with discrete spectrum;
- dissipation control in density matrix master equation;
- application for the simulation of the Heisenberg model.

During the second year I attended the following conferences and summer school:

- ”Statistical Mechanics & Field Theory”, Bari;
- ”Problemi Attuali di Fisica Teorica”, Vietri sul mare, with talk ”Bound states and entanglement generation in waveguide QED”;
- ”Information Geometry, Quantum Mechanics and Applications”, San Rufo-Policeta, with talk ”Correlated photon emission by two excited atoms in a waveguide”;
- ”Collective behavior in quantum matter”, summer school in ICTP (Trieste), with poster ”Correlated photon emission by two excited atoms in a waveguide”.