Università degli Studi di Bari Aldo Moro
Dipartimento Interateneo di Fisica "M. Merlin"
INFN Istituto Nazionale di Fisica Nucleare

Quantum Simulations for Lattice Gauge Theories

Domenico Pomarico

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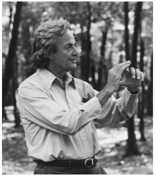
Quantum Simulator

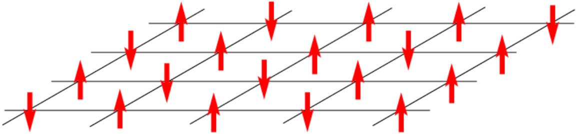


A quantum simulator is defined as a simple and experimentally feasible system that mimics a quantum system of interest.

N spins

$\downarrow$

2^N degrees of freedom



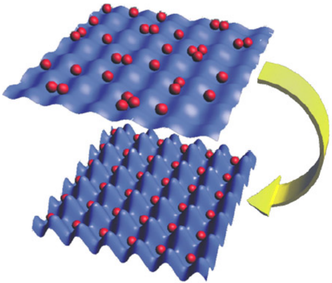
$\downarrow$

In a quantum simulator, the computational complexity of the simulation grows only polynomially with the number of particles.[1]

[1] **Feynman R. P.**, **International Journal of Theoretical Physics 21**, **467-488 (1982)**.

Controllable Quantum Systems

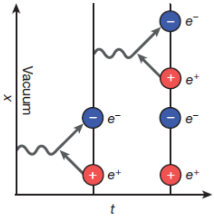
Laser fields are currently employed to create external potentials (e.g. optical lattices) and to engineer interactions between atoms in a controllable way. Quantum bits (qubits), representing the basis for quantum information processing, can be encoded in the states of atoms or ions.



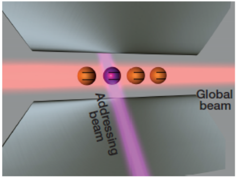
One of the most relevant successes of cold atom physics is the observation of the Mott insulator transition in an optical lattice.[2] This has been made possible by measurement techniques that enables one to address the state of single atoms.

[2] Greiner, Mandel, Esslinger, Hansch & Bloch, Nature 415, 39-44 (2002).

Experimental realization of a preliminary Quantum Simulator



The analysis of the vacuum quantum fluctuations is one of the most interesting topics in gauge theories.



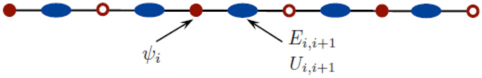
These effects have been studied in a preliminary quantum simulation of the $(1+1)$-dimensional quantum electrodynamics, implemented on four trapped ions. The experimental setup for the simulation consists of a linear Paul trap, where a string of $^{40}Ca^{+}$ ions is confined.[3]

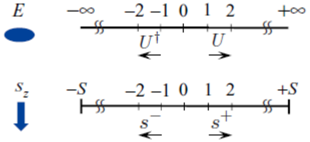
[3] Martinez, Muschik, Schindler, Nigg, Erhard, Heyl, Hauke, Dalmonte, Monz, Zoller & Blatt, Nature 534, 516-519 (2016).

(1+1)-dimensional lattice QED

A system of fermions defined on lattice sites (ψ_i) is coupled to a U(1) Abelian gauge field living on the links connecting neighboring sites (electric field $E_{i,i+1}$, gauge transporter $U_{i,i+1}$). The Gauss law selects the physical sector of the Hilbert space:

$$\psi_i^\dagger \psi_i + \frac{1}{2}[(-1)^i - 1] = (E_{i,i+1} - E_{i-1,i}).$$





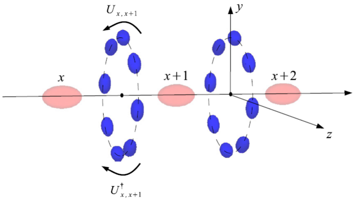
In the Quantum Link Model, gauge fields are represented by spin operators ($s^z_{i,i+1}, s^+_{i,i+1}$) applying a truncation of link Hilbert spaces.

Discrete Schwinger-Weyl Group

An alternative model that reduces the link Hilbert spaces to n dimensions and preserves the unitary gauge transporter is obtained by the discretization of the Weyl group. The electric field and the vector potential satisfy the commutation relation:

$$\left[E_{x,x+1}, A_{x',x'+1}\right] = i\delta_{x,x'},$$

which is the basis of the continuous Weyl group, formed by transformations $\{e^{-i(\eta A_{x,x+1}+\xi E_{x,x+1})}\}$, with $\eta, \xi \in \mathbb{R}$.



Considering as group parameter only $\ell \in \mathbb{Z}$, the discretization procedure chooses the electric field basis $\{|\nu_\ell\rangle\}_{1\leq\ell\leq n}$ and defines a unitary operator U such that $U|\nu_\ell\rangle = |\nu_{\ell+1}\rangle$ on the circle, obtaining a representation of the group $\mathbb{Z}_n$, with a possible implementation of links with ring-shaped transverse lattices.[4]

[4] Notarnicola, Ercolessi, Facchi, Marmo, Pascazio & Pepe, Journal of Physics A: Mathematical and Theoretical 48, 1-12 (2015).

Research Project

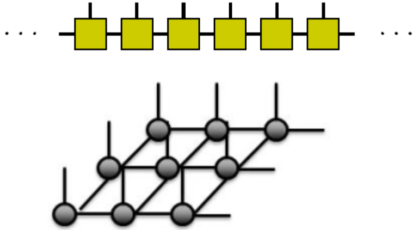
I will study the application of the discrete Schwinger-Weyl group to the (2+1)-dimensional QED. This extension is not trivial due to the following reasons:

- the Gauss law needs an adaptation with a new formulation;
- the Hamiltonian presents new terms related to the presence of the magnetic field, whose implementation can be hard.

Density Matrix Renormalization Group

The DMRG algorithm[5] is a numerical variational technique to analyze ground state properties of one-dimensional strongly correlated systems. This iterative procedure reduces the effective degrees of freedom to those relevant for the dynamics, limiting the exponential growth of the Hilbert space with the system size.

Computational complexity increases as the energy gaps become smaller. The DMRG method allows to describe also thermal states and can be generalized to more than one dimension.

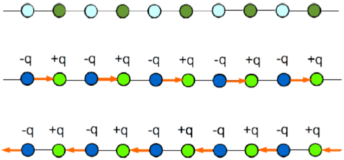


[5] White, PRL 69, 2863-2866 (1992).

Phase transitions in (1+1)-d Abelian gauge theories

The Hamiltonian of the $(1+1)$-dimensional lattice QED is:

$$H = -w \sum_x \left(\psi_x U^\dagger_{x,x+1} \psi^\dagger_{x+1} + H.c. \right) + m \sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{g^2}{2} \sum_x E^2_{x,x+1}.$$



The ground state can become doubly degenerate in certain parameter ranges.

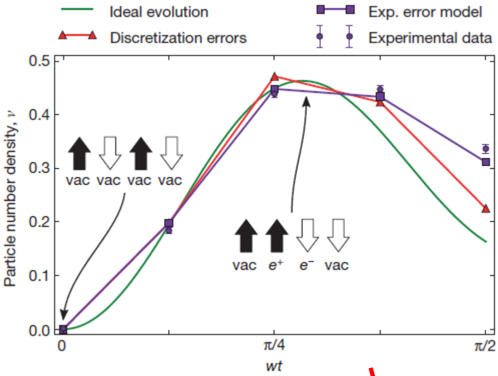
I will study phase transitions of lattice Abelian gauge theories in the variation of mass m and electric charge g, determining the phase diagram and identifying the universality class, with a detailed study of its connection with the Ising model.

Conclusions

The main topics of the research project are:

- application of the discrete Weyl-Schwinger group to the (2+1)-dimensional QED;
- investigation of ground state phase transitions, in mass and electric charge parameters, via a numerical approach based on the Density Matrix Renormalization Group;
- identification of universality classes in 1D (possible extension of the analysis in 2D).

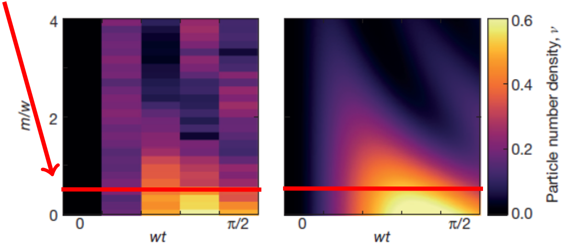
Experimental Data



$wt \longrightarrow$ dimensionless time

$w = 1/2a$ is the coupling constant for creation and annihilation of particle–antiparticle pairs, where a represents the lattice spacing

$m \longrightarrow$ particle mass

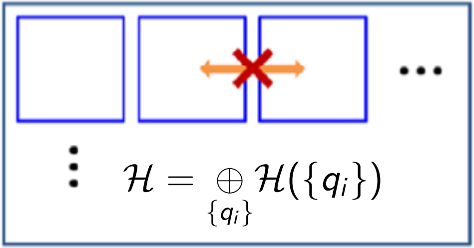


Physical states sector

The gauge-invariance condition for the Hamiltonian H reads:

$$[H, G_i] = 0,$$

thus dividing the Hilbert space $\mathcal{H}$ into sectors of eigenvalues of G_i.



Each such sector $\mathcal{H}(\{q_i\})$ has a set of eigenvalues $\{q_i\}$, called static charges, such that for every $|\psi(\{q_i\})\rangle \in \mathcal{H}(\{q_i\}) : G_i |\psi(\{q_i\})\rangle = q_i |\psi(\{q_i\})\rangle$.

In the defined G_i of the $(1+1)$-dimensional QED, the charge is absorbed into the generator, implying the physical subspace:

$$\mathcal{H}_G = \{|\psi\rangle \in \mathcal{H} : G_i |\psi\rangle = 0\},$$

while for the discrete Schwinger-Weyl group: $\prod_x e^{i\alpha_x G_x} |\psi\rangle = |\psi\rangle$, where α_x is a real function on the lattice.