



UNIVERSITÀ  
DEGLI STUDI DI BARI  
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**DOTTORATO IN FISICA XXXII CICLO**  
**Dipartimento di Fisica "Michelangelo Merlin"**

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# Dynamics of complex and active fluids

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**DOTTORANDO:**  
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# Summary

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- Complex and active fluids
- Dynamical model
- Lattice Boltzmann Method
- Activities

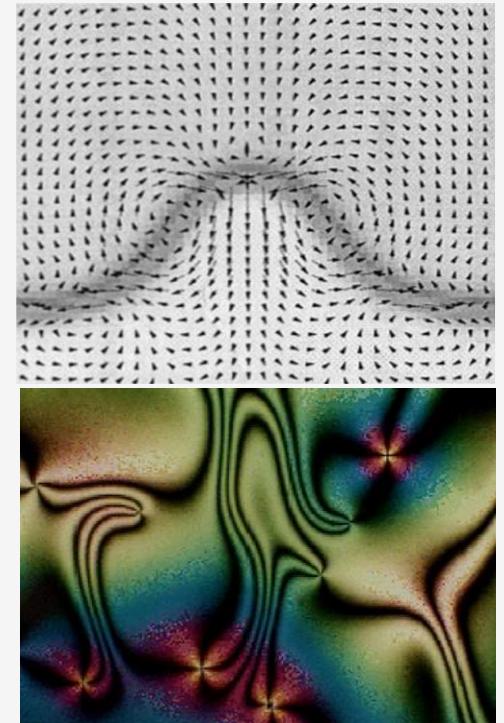
# Complex fluids

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with focus on the coupling between the velocity field and the defects dynamics.

Examples:

- **Binary mixture**  
velocity fields of different intensity and directions at interfaces
- **Liquid crystals**  
coupling between velocity field and orientation



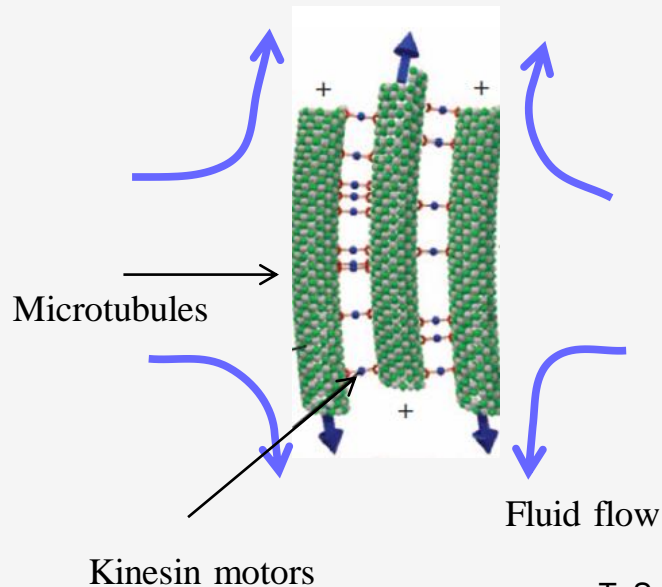
Every defect has a topological charge  $\longrightarrow$  flows of different intensity  
(G. Toth, C. Denniston, and J. M. Yeomans, "Hydrodynamics of Topological Defects in Nematic Liquid Crystals", Phys. Rev. Lett. 88, 105504 (2002).)

# Active systems

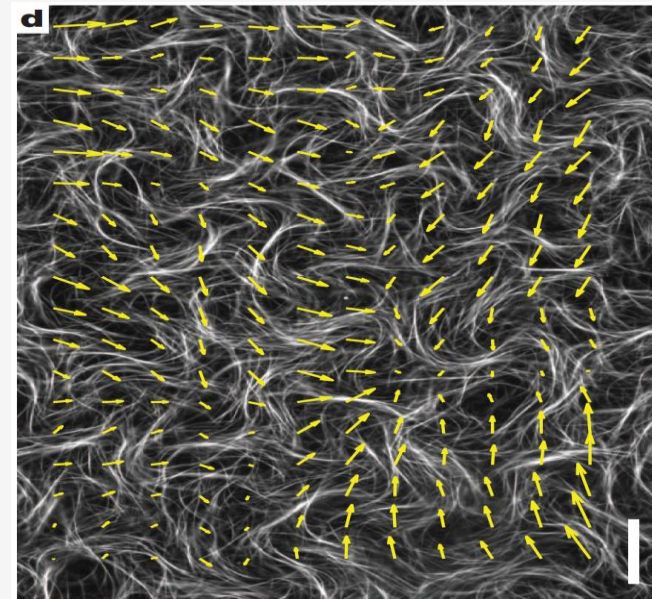
**Active matter** are self-driven systems which live, or function, far from equilibrium.

- **Interplay between interaction and activity**
- **High level of collective organization**

Extensile/pusher swimmer



Active microtubule network

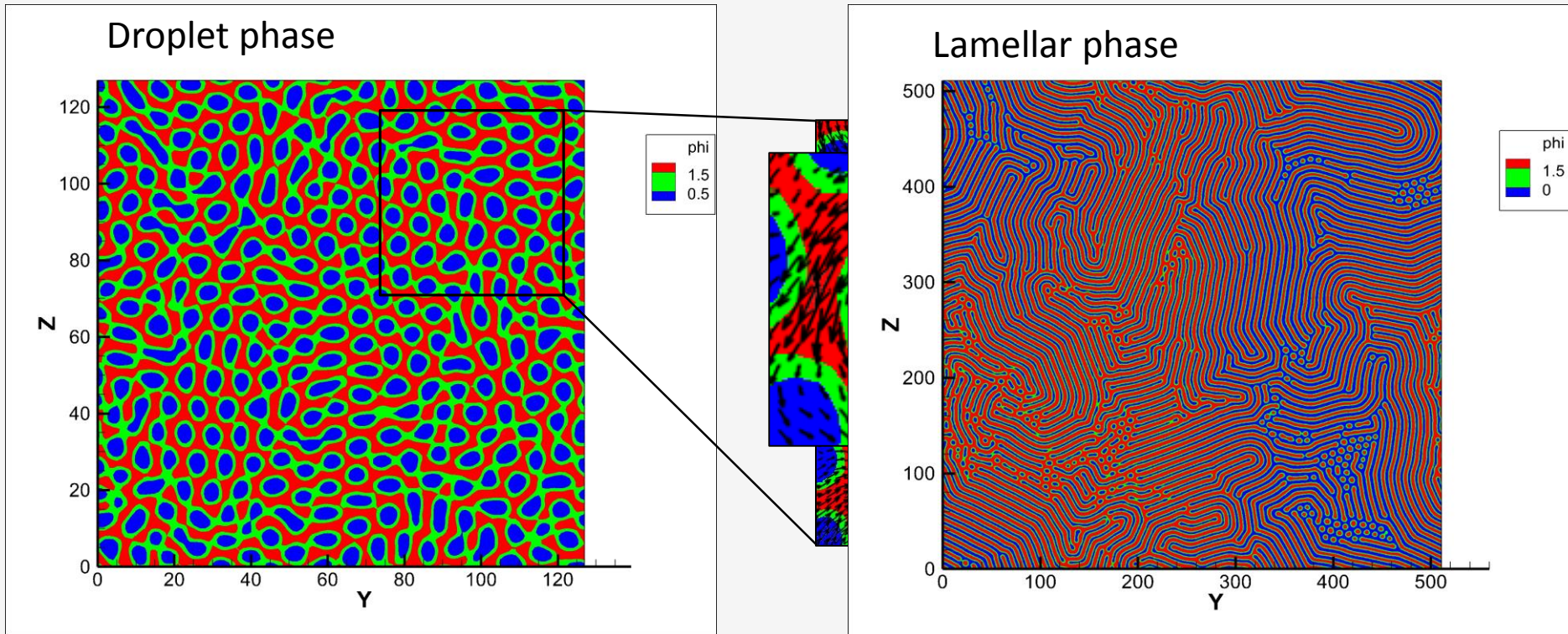


T. Sanchez et al., Nature, 491, 431 (2012)

# A new class of complex fluids: mixtures of active and passive components

Examples:

- Active droplets (M Blow et al., PRL 2014)
- Exotic emulsions (Gonnella, Tiribocchi, Bonelli, Orlandini, in



# Dynamical model

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Free energy based model (Ginzburg-Landau)

$$\mathcal{F} = \int dV \left( \frac{a}{2} \phi^2 + \frac{b}{2} \phi^4 + \frac{K}{2} |\nabla \phi|^2 \right)$$

$K$  linked to surface tension  $\tilde{\sigma} = \int dy \left( \frac{K}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \psi(\phi, T) - \psi_{\pm}(\phi, T) \right)$

$\psi(\phi, T)$  free energy density

$$\mu = \frac{\delta \mathcal{F}}{\delta \phi} = a\phi + b\phi^3 - K\nabla^2 \phi$$

$$\Pi_{\alpha\beta} = \left\{ \rho \frac{\delta \mathcal{F}}{\delta \rho} + \phi \frac{\delta \mathcal{F}}{\delta \phi} - \psi(\rho, \phi, T) \right\} \delta_{\alpha\beta} + D_{\alpha\beta}(\phi)$$

# Dynamical model

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$\eta$  dynamic viscosity

$\xi$  second viscosity coefficient

Continuity equation  $\partial_t \rho = -\partial_\alpha (\rho u_\alpha)$

Navier-Stokes  $\partial_t (\rho u_\alpha) + \partial_\beta (\rho u_\alpha u_\beta) = -\partial_\beta (\Pi_{\alpha\beta} - \sigma_{\alpha\beta})$

Viscous stress tensor  $\sigma_{\alpha\beta} = \eta \left( \partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2\delta_{\alpha\beta}}{d} \partial_\gamma u_\gamma \right) + \xi \delta_{\alpha\beta} \partial_\gamma u_\gamma + \zeta \left( P_\alpha P_\beta - \frac{1}{3} P^2 \delta_{\alpha\beta} \right)$

Convection-diffusion  $\partial_t \phi + \partial_\alpha (\phi u_\alpha) = -2\partial_\alpha J_\alpha^d$

## For liquid crystals

- Tensorial order parameter

## For active fluids

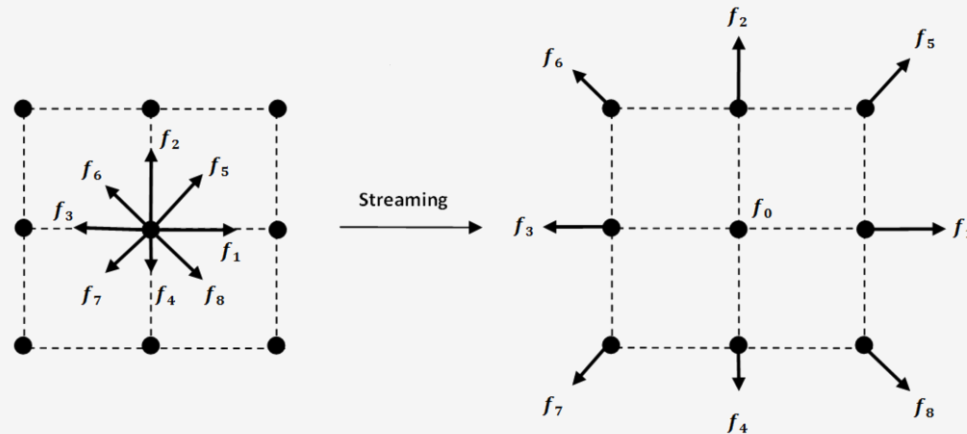
- Active stress tensor  
Ramaswamy et al PRL (2004)

# Lattice Boltzmann methods

Boltzmann transport equation

$$\frac{\partial f(\vec{x}, t)}{\partial t} + \vec{u} \cdot \nabla f(\vec{x}, t) = \Omega$$

“be wise, discretize!”  $f_i(\vec{x} + c \vec{e}_i \Delta t, t + \Delta t) - f_i(\vec{x}, t) = -\Delta t \frac{[f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)]}{\tau}$





# Lattice Boltzmann methods

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$$\rho(\vec{x}, t) = \sum_{i=0}^8 f_i(\vec{x}, t)$$

$$\vec{u}(\vec{x}, t) = \frac{1}{8} \sum_{i=0}^8 c f_i(\vec{x}, t) \vec{e}_i$$

The algorithm can be summarized as follows

1. Initialize  $\rho, \vec{u}, f_i$  and  $f_i^{eq}$
2. Streaming step: move  $f_i \rightarrow f_i^*$  in the direction of  $\vec{e}_i$
3. Compute macroscopic  $\rho$  and  $\vec{u}$  from  $f_i^*$
4. Compute  $f_i^{eq}$  in BGK approximation
5. Collision step: calculate the update distribution  $f_i = f_i^* - \frac{1}{\tau} (f_i^* - f_i^{eq})$
6. Repeat from step 2.

**In the continuum limit it is possible to derive Navier-Stokes equations from LBE**

# Activities

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- **Extend previous work on single component systems with phase transition**
  - Extend recent work[1] on liquid-vapor systems to systems of real dimensionality
  - Parallelization
  - Calculation of domain growth exponents in phase separation dynamics
  - Study of the Rayleigh-Plesset equation in  $d=3$
- **Mixtures of active and passive fluids (2d)**
  - Characterization of different phases and dynamics of transition
  - Response to external flows

[1]Pattern formation in liquid-vapor systems under periodic potential and shear  
A Coclite, G Gonnella, A Lamura - Physical Review E, 2014

