UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO
DIPARTIMENTO INTERATENEO DI FISICA "M. MERLIN"
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Quantum Simulations for Lattice Gauge Theories

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Corso di Dottorato XXXII ciclo - Relazione attività di ricerca I anno

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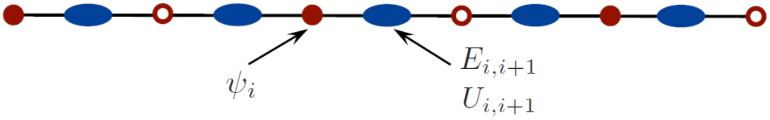
Schwinger Model

$$\mathcal{H} = -\frac{1}{2a}\sum_i \left(\psi_i U^\dagger_{i,i+1}\psi^\dagger_{i+1} + H.c.\right) + m\sum_i (-1)^i \psi^\dagger_i \psi_i + \frac{g^2 a}{2}\sum_i E^2_{i,i+1},$$

where $\{\psi_i, \psi_j^\dagger\} = \delta_{i,j}$, $U_{i,i+1} = e^{-iA_{i,i+1}}$, $[E_{i,i+1}, A_{j,j+1}] = i\delta_{i,j}$.



$$\psi^\dagger_i \psi_i + \tfrac{1}{2}[(-1)^i - 1] = E_{i,i+1} - E_{i-1,i}$$

Gauss law selects the physical subspace

number operator — staggered vacuum — electric field divergence

Dirac Equation

In the most general setting:

- Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$ for $N \times N$ matrices γ^μ with $\mu = 0, 1, \ldots, D-1$

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$$SO(D-1,1) \text{ invariance} \quad \longrightarrow \quad \text{spinorial representation}$$

$$\begin{cases} \frac{(D-1)(D-2)}{2} \text{ independent rotations} \\ D-1 \text{ independent boosts} \end{cases}$$

Chiral representation exists only for even D.

I studied free Dirac equation in $D = 2$ implementing the boost of rest-frame positive and negative energy solutions, the interacting case and the absent chiral representation in $D = 3$ will be considered.

Discrete Heisenberg Group

Continuous Heisenberg group with 2 parameters $\alpha, \beta \in \mathbb{R}$:

$$U(\alpha) = e^{-i\alpha A}, \quad V(\beta) = e^{i\beta E}, \quad U(\alpha)V(\beta) = e^{-i\alpha\beta} V(\beta)U(\alpha).$$

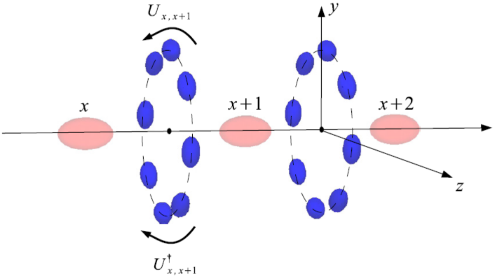
Electric field representation:

$$V(\beta)\varphi(E) = e^{i\beta E}\varphi(E),$$
$$U(\alpha)\varphi(E) = \varphi(E+\alpha)$$

Discretization:

$$\beta \to \sqrt{\frac{2\pi}{n}}\,\ell, \ell \in \mathbb{Z}$$

$$\begin{cases} U\,|E_\ell\rangle = |E_{\ell+1}\rangle \text{ for } \ell < n, \\ U\,|E_n\rangle = |E_1\rangle \end{cases}$$



Non-Abelian Gauge Theories

$SU(2)$, in a Yang-Mills matrix model:

$$A_i = A_{ia}\frac{\sigma_a}{2}, \quad E_j = E_{jb}\frac{\sigma_b}{2} \quad \text{with} \quad [A_{ia}, E_{jb}] = i\delta_{ij}\delta_{ab},$$

where $[\sigma_a, \sigma_b] = 2i\epsilon_{abc}\sigma_c$, with $i, j = 0, \ldots, D-1$ in D dimensions.

The search for a closed algebra for gauge variables requires calculations of:

- $U_i = e^{-iA_i}$
- $[U_i, E_j]$, extended to the more general $[e^{-i\alpha_i A_i}, \beta_j E_j]$

To proceed towards discretization, the most general commutator between $e^{-i\alpha_i A_i}$ and $e^{i\beta_j E_j}$ will be obtained.

Quantum Computing

$$N \text{ qudit} : \mathscr{H} = \bigotimes_{i=1}^{N} \mathscr{H}_i \quad \Longrightarrow \quad \dim \mathscr{H} = d^N$$

SCALABLE QUANTUM SYSTEMS

- computing power enhancement proportional to the number of basis states in superposition

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SCALABLE QUANTUM SYSTEMS

- computing power enhancement proportional to the number of basis states in superposition
- computational complexity for simulations

To manage the computational resource request in a simulation a polynomial growth for the number of degrees of freedom is needed:

Singular value decomposition (SVD) applied to coefficients arrays reshaped in matrices ⇒ entanglement cut-off

Matrix Product States

Linear lattice with L sites: $|\psi\rangle = \sum_{\boldsymbol{\sigma}} c_{\sigma_1 \ldots \sigma_L} |\boldsymbol{\sigma}\rangle$, $|\boldsymbol{\sigma}\rangle = |\sigma_1\rangle \otimes \cdots \otimes |\sigma_L\rangle$

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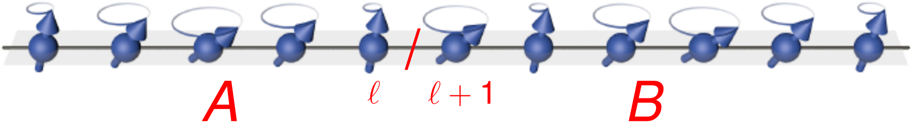
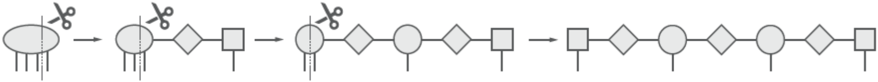


SVD: $|\psi\rangle = \sum_{\boldsymbol{\sigma}} \sum_{a_\ell} A^{(\sigma_1 \ldots \sigma_\ell)}_{1,a_\ell} S_{a_\ell,a_\ell} B^{(\sigma_{\ell+1} \ldots \sigma_L)}_{a_\ell,1} |\boldsymbol{\sigma}\rangle = \sum_{a_\ell} s_{a_\ell} |a_\ell\rangle_A |a_\ell\rangle_B$

Iterating site by site:

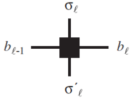


$$|\psi\rangle = \sum_{\boldsymbol{\sigma}} \sum_{a_1,\ldots,a_{L-1}} A^{\sigma_1}_{1,a_1} \ldots A^{\sigma_\ell}_{a_{\ell-1},a_\ell} S_{a_\ell,a_\ell} B^{\sigma_{\ell+1}}_{a_\ell,a_{\ell+1}} \ldots B^{\sigma_L}_{a_{L-1},1} |\boldsymbol{\sigma}\rangle ,$$

a MPS, with a possible cut-off over singular values s_{a_ℓ}.

Tensor Network Operators

Matrix Product Operators (MPO) for linear lattice Hamiltonians:



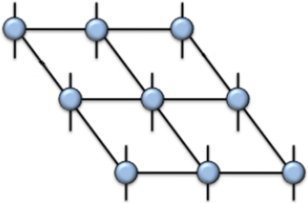
$\longrightarrow$ operator-valued matrices $W^{[\ell]\sigma_\ell,\sigma'_\ell}_{b_{\ell-1},b_\ell}$

$$\mathcal{H} = W^{[1]} W^{[2]} \ldots W^{[L]}$$

open boundary conditions

Periodic boundary conditions require 2 different matrices:

$W^{[1]}, W^{[i]}$ $(2 \leq i \leq L)$ $\longrightarrow$ a single matrix requires an increased order



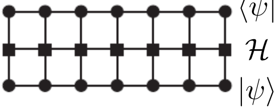
Projected Entangled Pair Operator can model Hamiltonians for lattices in more dimensions

$\longrightarrow$ I built a cylindrical and toroidal plaquette, 2×3 lattice and torus, 3×3 lattice, cylinder and torus.

Tensor Network Theory

Ground state research:

$$\langle\psi|\mathcal{H}\psi\rangle =$$



$\longrightarrow$ generalized eigenvalue problem

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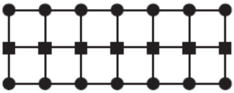


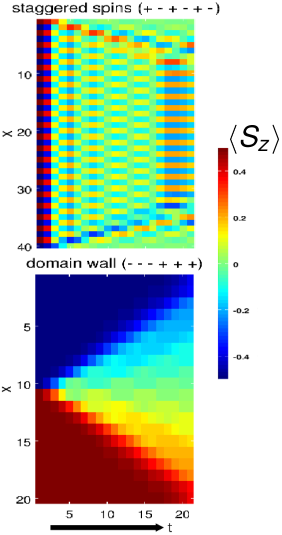
$\longrightarrow$ generalized eigenvalue problem

Time evolution:

Nearest-neighbor interaction
Suzuki-Trotter expansion for the propagator:

$$e^{i(\mathcal{H}_1+\mathcal{H}_2)t} = \lim_n (e^{i\mathcal{H}_1\frac{t}{n}} e^{i\mathcal{H}_2\frac{t}{n}})^n$$

implemented in TNT with a sweep over the linear lattice.



Outlook

Main activities in progress for the second year are:

- investigation of the consequences of the absent chiral representation in odd number of dimensions;
- discretization of non-Abelian gauge groups;
- application of the tensor network scheme in more than one dimension.