Buond states in the continuum in one-dimensional qubit arrays

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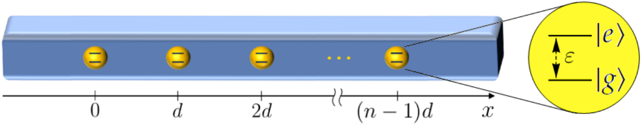
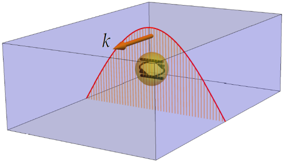
Corso di Dottorato XXXII ciclo - Relazione attività di ricerca

15 Ottobre 2019

Supervisors: Prof. Saverio Pascazio, Dott. Francesco V. Pepe

Experimental platform: Waveguide QED





- n two-level emitters with spacing d;
- structured 1D photon continuum with $\omega(k) = \sqrt{k^2 + m^2}$.

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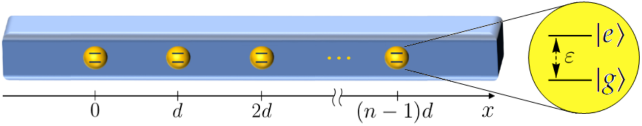

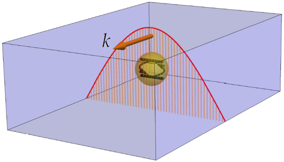

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$$H = H_0 + H_{\text{int}}, \quad \begin{cases} H_0 = \varepsilon \sum_{j=1}^{n} |e_j\rangle\langle e_j| + \int \mathrm{d}k\, \omega(k) b^\dagger(k) b(k) \\ H_{\text{int}} = \sum_{j=1}^{n} \int \mathrm{d}k \left[F(k)\, \mathrm{e}^{\mathrm{i}(j-1)kd} |e_j\rangle\langle g_j| b(k) + h.c. \right] \end{cases}$$

with $[b(k), b^\dagger(k')] = \delta(k - k')$ and form factor $F(k) = \sqrt{\frac{\gamma}{2\pi\omega(k)}}$. $\gamma \leftarrow$ coupling constant

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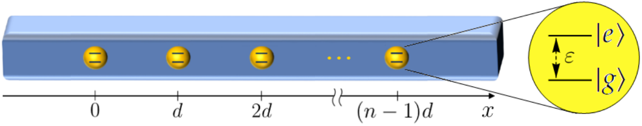

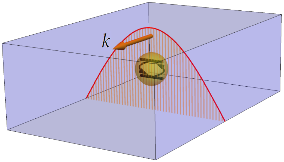

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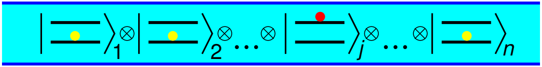
One excitation sector:

$$|\Psi\rangle = \sum_{j=1}^{n} a_j \, |E_j^{(n)}\rangle \otimes |\mathrm{vac}\rangle + |G^{(n)}\rangle \otimes \int \mathrm{d}k \, \xi(k) b^\dagger(k) \, |\mathrm{vac}\rangle = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ |\xi\rangle \end{pmatrix}$$

with $|E_j^{(n)}\rangle =$ 

Eigensystem: $n = 3$

Inverse propagator: $G^{-1}(z) = (z - \varepsilon)\mathbb{1} - \Sigma(z)$

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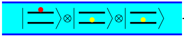
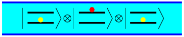
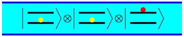
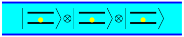
$G^{-1}(E)\mathbf{a} = 0$ with global state:

$|\Psi\rangle = a_1$  $+ a_2$ 

$+ a_3$  $+$  $\otimes |\xi\rangle$

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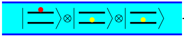
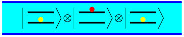
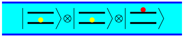
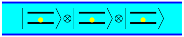
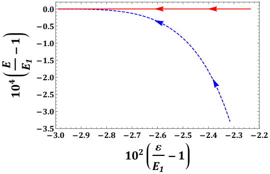
$|\Psi\rangle = a_1$  $+ a_2$ 

$+ a_3$  $+$  $\otimes |\xi\rangle$

Bound states

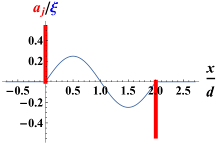


Resonance eigenvalues

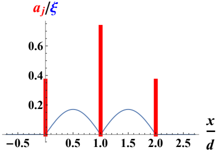
$$E_\nu(d) = \sqrt{\frac{\nu^2\pi^2}{d^2} + m^2}$$

Eigenstates components

$$\frac{a_1}{a_3} = -1 \qquad a_2 = 0$$



$$\frac{a_1}{a_3} = 1 \qquad a_2 \approx 2$$



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Eigensystem: $n = 4$

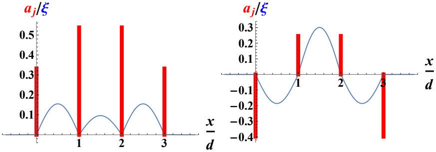
excitation amplitudes

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \in \mathbb{R}^4$$

$$\xi \in L^2(\mathbb{R})$$

photon wavefunction

Eigenstates components



$$\frac{a_1}{a_4} = \frac{a_2}{a_3} = 1, \quad \frac{a_1}{a_2} \approx \frac{\sqrt{5}-1}{2}$$

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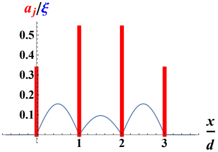

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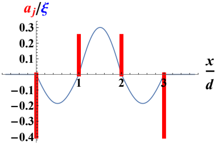

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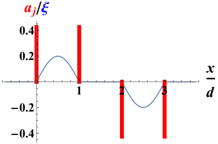

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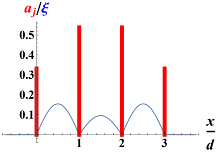

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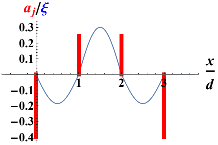

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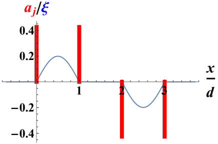

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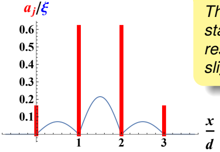
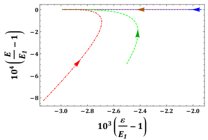


$$\frac{a_1}{a_4} = \frac{a_2}{a_3} = 1, \quad \frac{a_1}{a_2} \simeq 0.25$$

Bound states



New lowest-energy BIC

The energy of the emerged state for $n = 4$ is shifted with respect to the resonant values, slighlty smaller than $E_1(d)$.

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Graph Laplacian self-energy matrix

$$\mathcal{U}_n \Sigma_n(\varphi) \mathcal{U}_n^\dagger = \Sigma^+_{\lfloor n/2 \rfloor}(\varphi) \oplus \Sigma^-_{\lceil n/2 \rceil}(\varphi)$$

with $\mathcal{U}_n$ given by Toeplitz-Hankel eigenvectors central symmetry

$n = 2h + 1$

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$$\Sigma^-_h(\varphi) = \begin{pmatrix} 2\varphi & 1 & & & \\ 1 & 2\varphi & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2\varphi & 1 \\ & & & 1 & 2\varphi \end{pmatrix}$$

Dirichlet-Dirichlet boundary: $a_0 = a_{h+1} = 0$

$$\lambda_j = -4\sin^2\left(\frac{j\pi}{2(h+1)}\right)$$

$$a^{(j)}_\ell = \sqrt{\frac{2}{h+1}} \sin\left(\frac{j\pi}{h+1}\ell\right)$$

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Dirichlet-(anti)Neumann boundary: $\begin{array}{l} a_0 = 0 \\ a_h = \pm a_{h+1} \end{array}$

$$\lambda_j = \mp 4 \sin^2\left(\frac{(j - 1/2)\,\pi}{2h+1}\right)$$

$$a_\ell^{(j)} = \sqrt{\frac{2}{h+\frac{1}{2}}} (\pm 1)^{\ell+1} \sin\left(\frac{\left(j - \frac{1}{2}\right)\pi}{h + \frac{1}{2}}\ell\right)$$

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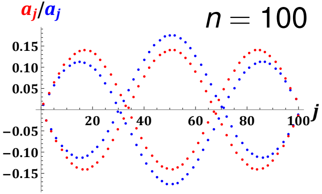
Rank-one modification of the symmetric eigenproblem

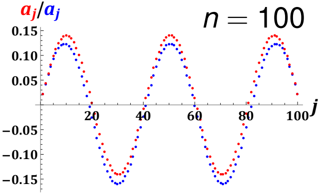
$$\widetilde{\Sigma}_{\lceil n/2\rceil}(b) = b\Sigma_{\lceil n/2\rceil}(\varphi) - n\,\mathrm{i}\,\mathbf{u}\otimes\mathbf{u}^T \quad \text{with eigenvectors } \widetilde{\mathbf{a}}^{(j)}$$

$$\Sigma_{\lceil n/2\rceil}(\varphi) = \Xi\Lambda\Xi^T \Longrightarrow \widetilde{\Sigma}_{\lceil n/2\rceil}(b) = \Xi\left(b\Lambda - n\,\mathrm{i}\;\mathbf{z}\otimes\mathbf{z}^T\right)\Xi^T = \widetilde{\Xi}\,\widetilde{\Lambda}\,\widetilde{\Xi}^T$$

$\boxed{\nu = 1}$

$\widetilde{\mathbf{a}}^{(1)}$





$\boxed{\nu = 2}$

$\widetilde{\mathbf{a}}^{(2)}$

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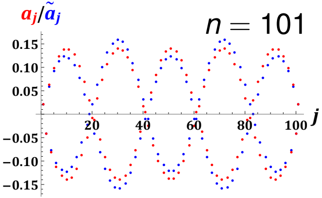

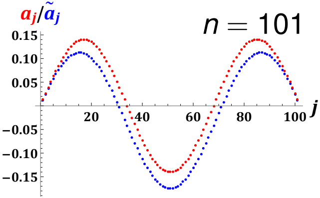

$\widetilde{\mathbf{a}}^{(1)}$

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Determinant factorization: $n = 2h + 1$

$$G^{-1}(E) = -\frac{\mathrm{i}\gamma}{\sqrt{E^2 - m^2}} A_n(E), \quad \det A_h^-(E) = U_h(E)$$

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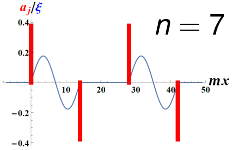
Chebyshev polynomial
of the second kind



- the eigenstate is composed of two trimers

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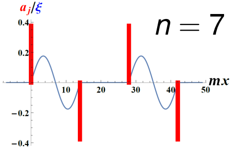

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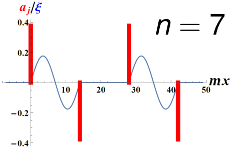
Chebyshev polynomial of the second kind

$n = 7$



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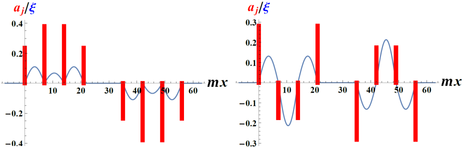
$n = 9$



- tetramerized system

$$\det A_9^-(E) = \det A_4^-(E) \det A_4^+(E)$$

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Conclusions & Outlook

- Fermi's golden rule: $|\Psi(0)\rangle = \begin{pmatrix} \mathbf{a}^{(j)} \\ |\xi\rangle \end{pmatrix} \quad \longrightarrow \quad |\Psi(t)\rangle$?

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$$\begin{pmatrix} \Xi \\ \widetilde{\Xi} \end{pmatrix} \begin{pmatrix} G & 0 \\ 0 & \widetilde{G} \end{pmatrix} \begin{pmatrix} \Xi^T & \widetilde{\Xi}^T \end{pmatrix} = \mathrm{diag}\left(\left\{ \frac{1}{E - \varepsilon - \Delta_{\mathbf{a}^{(j)}} + \frac{\mathrm{i}}{2}\Gamma_{\mathbf{a}^{(j)}}} \right\}_j \right)$$

Breit-Wigner expansion: $\Gamma_{\mathbf{a}^{(j)}}(\varepsilon) = 2\pi \boxed{\kappa_{\mathbf{a}^{(j)}}(\varepsilon)} \leftarrow$ spectral density

Experimental study of dark states, with the aim of implementig a noise-free memory able to represent a quantum register;

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- High-energy eigenstates:

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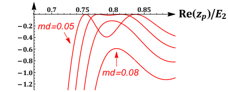
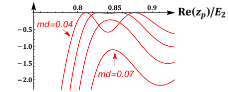


$n = 4$



k nearest-neighbors study for a high qubits number;

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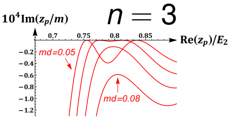
- Fermi's golden rule: $|\Psi(0)\rangle = \begin{pmatrix} \mathbf{a}^{(j)} \\ |\xi\rangle \end{pmatrix} \quad \longrightarrow \quad |\Psi(t)\rangle$?

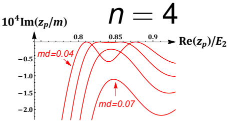
$$\begin{pmatrix} \Xi \\ \widetilde{\Xi} \end{pmatrix} \begin{pmatrix} G & 0 \\ 0 & \widetilde{G} \end{pmatrix} \begin{pmatrix} \Xi^T & \widetilde{\Xi}^T \end{pmatrix} = \mathrm{diag}\left(\left\{ \frac{1}{E - \varepsilon - \Delta_{\mathbf{a}^{(j)}} + \frac{\mathrm{i}}{2}\Gamma_{\mathbf{a}^{(j)}}} \right\}_j \right)$$

Breit-Wigner expansion: $\Gamma_{\mathbf{a}^{(j)}}(\varepsilon) = 2\pi \boxed{\kappa_{\mathbf{a}^{(j)}}(\varepsilon)} \leftarrow$ spectral density

Experimental study of dark states, with the aim of implementig a noise-free memory able to represent a quantum register;

- High-energy eigenstates:





k nearest-neighbors study for a high qubits number;

- Multimode Waveguide QED: implementation of qubit arrays coupled with more than a single mode.

During the third year I attended the following conferences:

- ”Problemi Attuali di Fisica Teorica”, Vietri sul mare;
- ”The 51st Symposium on Mathematical Physics”, Torun (Poland), with poster ”Bound states in the continuum for an array of quantum emitters”;
- ”Italian Quantum Information Science conference”, Milano, with poster ”Correlated photon emission by two excited atoms in a waveguide”.

Publications list:

- P. Facchi, M. S. Kim, S. Pascazio, F.V. Pepe, D. Pomarico, and T. Tufarelli, *Phys. Rev. A* **94**, 043839 (2016);
- P. Facchi, S. Pascazio, F.V. Pepe, and D. Pomarico, *Phys. Rev. A* **98**, 063823 (2018);
- P. Facchi, D. Lonigro, S. Pascazio, F.V. Pepe, and D. Pomarico, *Phys. Rev. A* **100**, 023834 (2019);
- P. Facchi, D. Lonigro, S. Pascazio, F.V. Pepe, and D. Pomarico, “Spin waves and multimerization for many-body bound states in the continuum”, in preparation.